SOUND RADIATION FROM FLUID-COUPLED STRIP PLATES WITH LONGITUDINAL STRIFFENERS

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In large structures like ships, trains, etc., stiffened single or double layered plates are widely used so that it is important to understand the vibrating and radiating behavior of these plate structures, especially coupled with fluids. In this study, vibration and sound radiation characteristics of strip plates which have finite width and infinite length are investigated numerically regarding longitudinal stiffeners and fluid coupling. The waveguide finite element and boundary element approach is used in this purpose. First of all, for a simple strip plate, the effects of stiffeners and fluid coupling are investigated in terms of the dispersion diagrams. Then the radiated sound power and radiation efficiency of the stiffened plate fluid coupled are calculated and compared to see the effect of the stiffeners on radiated power of strip plates.

1. Introduction

Many large structures like ships, offshore platforms, etc., in marine engineering fields are built up with single and/or double layered plates with complicate stiffeners for their structural strength. Also the underwater radiated noise from ships becomes interested more and more in these days. Therefore, in order to predict the structural and acoustical responses of these structures, it is necessary to understand the vibration and radiation characteristics of these stiffened plates. These plates are often connected with heavy fluids like water, oil, etc. In ships, for example, the hull plates are normally coupled with external and/or internal fluids. In these conditions, the fluid coupling needs to be regarded to predict structural responses and internal/external noise problems in ships. So this work aims to investigate the variation of radiation characteristics of the plate with and without the fluid coupling and stiffeners.

To make the problem simpler, the stiffened plates are simplified as strip plates which have uniform cross-sections along their infinite lengths in this study. In this work, vibration characteristics and sound radiations of infinite length strip plates are investigated numerically by means of the waveguide finite element and boundary element (WFE/BE) method. The WFE/BE method models only 2D cross-sections of the waveguide structures and fluids but takes into account the three dimensional nature of the infinite extent of the waveguide. In this numerical analysis, the longitudinal stiffeners are combined with strip plates to examine the effects of stiffeners on the wave propagation and sound radiation.

The plates regarded in this analysis are stiffened single and double layered plates. First of all, the effects of stiffeners and fluid coupling are investigated in terms of the dispersion relations to examine the types of waves sustained in the strip plates. Next, the forced structural vibration and radiated noise are calculated for the purpose of predicting the radiation efficiency. In order to disre-
garding the cross-modal terms, the time and space averaged vibration level and radiated sound powers are predicted from the WFE/BE method [1].

For the prediction of radiated noise from ships, the ship hull plates are frequently divided into many subplates by the span of stiffeners. Then the radiation efficiencies of the subplates are multiplied to the each plate’s vibration level to obtain the radiated sound power from the subplates. In this study, this prediction scheme is to be examined by comparing the radiated powers from whole plate and the integrated one from each subdivided plate.

2. Waveguide finite element and boundary element approach for fluid-coupled strip plates

In this section, a numerical method called waveguide finite element and boundary element (WFE/BE) method is applied to the strip plates with stiffeners. The WFE method models only 2D cross-sections of the waveguide structures but takes into account the 3D nature of the infinite extent of the waveguide. If the WFE model is contacted with a fluid, this fluid can be considered with BEs. The coupled WFE/BE method can be used to investigate sound radiated from the waveguide structures connected to the fluid. The WFE/BE method is briefly described here and details for this method refer to refs [2~5].

The WFE equation with plate elements and boundary elements is given by

\[
\{K_4(-ik_x)^4 + K_2(-ik_x)^2 + K_1(-ik_x) + K_0 - i\omega M\}\vec{\Phi} - i\omega \rho C_1\vec{\Psi} = \vec{F}, \quad (1)
\]

where \(k_x\) is the \(x\) directional wavenumber, \(K_4, K_2, K_1\) and \(K_0\) are the matrices that come from the stiffness of the structure, \(M\) is the mass matrix, \(C_1\) is the coupled matrix between WFEs and BEs and \(\vec{F}\) is the vector of excitation forces. \(\vec{\Phi}\) and \(\vec{\Psi}\) are the displacement vector and velocity potential vector, respectively, representing shapes of cross-sectional deformation. If a fluid contacted to the WFE model is light like air, the coupled term in the left-hand side in Eq. (1) can be disregarded. The continuity condition at the coupled dofs and the velocity potential, \(\vec{\Psi}\), and the velocity, \(\partial \vec{\Psi}/\partial n\), at the coupled fluid boundary are given by

\[
I_2 \frac{\partial \vec{\Psi}}{\partial n} = i\omega \rho C_2 \vec{\Phi}, \quad H\vec{\Psi} - G \frac{\partial \vec{\Psi}}{\partial n} = 0, \quad (2)
\]

where \(n\) is a normal directional vector of BEs connected with WFEs, \(I_2\) is an identity matrix and \(C_2\) is a matrix allocating fluid coupled dofs in \(\vec{\Phi}\). \(H\) and \(G\) are matrices of Green’s function. By solving Eqs (1) and (2), sound pressure \(\tilde{p}(k_x)\) and normal directional particle velocity \(\tilde{v}(k_x)\) at the coupled boundary are determined by

\[
\tilde{p}(k_x) = j\omega \rho_0 \tilde{\Psi}, \quad \tilde{v}(k_x) = \frac{\partial \vec{\Psi}}{\partial n} \quad (3)
\]

In order to evaluate the radiation efficiency, the radiated sound power and averaged mean-squared velocity of the plate are required. The sound power radiated from the structure can be obtained from the integration in wavenumber domain as [3]

\[
W = \frac{1}{4\pi} \text{Re} \left( \int_{-k}^{k} \int_{\Gamma} \tilde{p}^*(k_x) \tilde{v}(k_x) \, d\Gamma \, dk_x \right) \quad (4)
\]

where \(^*\) denotes the complex conjugate and \(\Gamma\) is the perimeter of the cross-section, contacted with fluid. The average mean-squared velocity of the plate can be obtained by
\[ \langle v^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int \frac{|\tilde{v}(k_x)|^2}{2} d\Gamma dk_x \] (5)

In the calculation of Eq. (5), the integration range is restricted from \(-k_{x,max}\) to \(k_{x,max}\) which is fairly bigger than the largest propagating wavenumber \(k_x\) at a frequency \(\omega\), i.e., \(k_{x,max} > \max (\text{Re}(k_x))\). From sound power and average mean-squared velocity obtained from Eq. (4) and Eq. (5), radiation efficiency can be found by

\[ \sigma = \frac{W}{pc\langle v^2 \rangle} \] (6)

3. Stiffened strip plates coupled with fluid

In this section, fluid coupled strip plates with longitudinal stiffeners are examined by the WFE/BE method to evaluate their radiation characteristics. The system of interest is conceptually illustrated in Fig. 1. The plate and stiffeners have infinite lengths along the \(x\) axis. The base plate is simply supported at both ends of \(y = 0\) and \(l_y\), and the outside of the plate, region \(y < 0, y > l_y\) are rigidly baffled. It is supposed in this model that the lower side of the plate is contacted with water.

The cross-sectional FE model of the plate with three stiffeners evenly spaced is shown in Fig. 2(a) and the properties are listed in Table 1. In this section, an unstiffened plate and a plate with single stiffener are also regarded to see the effects of the stiffeners on the vibration of the base plate and sound radiation. In addition, a double layered strip plate which has the upper plate on the stiffened plate is modelled as shown in Fig. 2(b). The properties of the upper plate are identical as the base plate and the cavities located between the two plates are supposed to be vacuous. The rest conditions remain the same as shown in Fig. 1.

![Figure 1](image1)

**Figure 1.** Schematic diagram for the stiffened strip plate coupled with water.

![Figure 2](image2)

**Figure 2.** Cross-sectional models for (a) the stiffened strip plate, (b) the stiffened double layered strip plate.
Table 1. Dimensions and material properties of the strip plate.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, $E$</td>
<td>210 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.332</td>
</tr>
<tr>
<td>Thickness of base plate, $h$</td>
<td>6 mm</td>
</tr>
<tr>
<td>Damping loss factor, $\eta$</td>
<td>0.1</td>
</tr>
<tr>
<td>Width of base plate, $l_y$</td>
<td>1 m</td>
</tr>
<tr>
<td>Density, $\rho$</td>
<td>7800 kg/m$^3$</td>
</tr>
<tr>
<td>Speed of sound in water</td>
<td>1500 m/s</td>
</tr>
</tbody>
</table>

An external force is applied vertically on the upper side of the plate. In this study, the input point force will be shifted along the $y$ axis to get averaged response over force point locations and assumed to be applied on the base plate, rather than on top of the stiffeners. For the double layered plate which has the upper plate, the force is regarded to act on the upper plate, rather than the bottom plate.

3.1 Dispersion relations

For the base plate without stiffeners, first of all, the dispersion diagrams are compared in Fig. 3(a) with and without fluid coupling. The dispersion diagram in Fig. 3(a) shows that many flexural waves are sustained in the base plate which possess the sinusoidal bending modes in the $y$ direction. Also it can be seen that in-plane waves which have high phase speeds are cut-on consecutively as frequency increases. In the case of fluid coupling, Fig. 3(a) shows that the cut-on frequencies of the flexural waves shift down to low frequencies due to the mass added by the coupled fluid. On the contrary, it is clearly seen that the in-plane waves are little affected by the fluid coupling in Fig. 3(a).

The dispersion curves of the plate with three stiffeners are shown in Fig. 3(b). It was reported in ref. [5] that stiffeners can be simplified as a smeared mass at low wavenumbers while work as rigid boundaries at high wavenumbers. In between the low and high wavenumber ranges, they govern the bending stiffness of the plate and then can be regarded as an equivalent beam. Hence each dispersion curve in Fig. 3(b) consists of three regions of different tendency of the slopes as wavenumber increases. Also it can be seen from Fig. 3(b) that the four dispersion curves are grouped together as the wavenumber increases. Each group contains four waves in general, because the plate with three stiffeners has four ‘bays’, which are the segments of plate separated by the two adjacent stiffeners. Note that there is an exceptional group (the third group) in Fig. 3(b) which has three curves. They represent lateral bending waves propagating predominantly through the three

![Figure 3](image_url)
Figure 4. Dispersion diagrams for the stiffened double layered plate with and without fluid coupling (a) in linear scale, (blue lines: fluid uncoupled; red dots: fluid coupled), (b) in logarithmic scale (dash line: acoustic wave in water).

When fluid is coupled to the stiffened plate, the dispersion curves move down to low frequencies and move up to high wavenumbers due to the added fluid mass. However, Fig. 3(b) implies that the waves propagating through the stiffeners are relatively not much affected by the fluid coupling of the base plate.

Acoustic waves in air and water are also plotted in Fig. 3 to reveal the wavenumber ranges in which sound radiates well, so called supersonic range. In Fig. 3, the structural waves which have less wavenumbers than that of the acoustic waves contribute to the far-field sound radiation. As shown in Fig. 3, the acoustic wave in water has about 4 times faster than that in air so that the supersonic wavenumber range becomes a quarter of that in air at each frequency. Therefore, in case of the fluid coupled plate, the less flexural waves contribute to sound radiation.

Dispersion curves of the stiffened double plate are shown in Fig. 4. The features of the wave grouping are observed again from Fig. 4 due to the presence of the stiffeners. In case of fluid uncoupled, each group now consists of eight waves corresponding to the number of bays (four each in upper and lower plates). Since the stiffeners are attached between the lower and upper plates, the waves propagating dominantly along the stiffeners will cut-on at fairly high frequencies and then not shown in Fig. 4. In the fluid coupled situation, only the bottom base plate is wet so that the eight waves in each group are separated into two groups of four waves in Fig. 4. The waves in the bottom plate are shifted down to the low frequencies but those in the upper plate remain because they are not much affected by the fluid coupling of the bottom plate. Fig. 4(b) is the same dispersion curves as Fig. 4(a) but with a logarithmic scale. In Fig. 4(b), the cut-on frequencies at low frequencies are more clearly seen. Fig. 4(b) is helpful in comparison with the forced responses of the double plate in Sec. 3.2.

### 3.2 Characteristics of sound radiation

In this section, the sound radiation from the stiffened strip plates is calculated regarding the fluid coupling. The forced responses of the plate depend upon the excitation location. In this study, the responses are calculated by shifting the excitation force for all the possible excitation positions along the \( y \) axis and then averaged. From the averaged mean-squared velocity of the plate and averaged sound power, the radiation efficiency is obtained as given in Eq. (6). This averaging over all possible forcing positions on the plate makes the cross-modal contribution become negligible [1].

Averaged mean-squared velocities for the unstiffened, stiffened and double layered plates are compared in Fig. 5(a). It can be found from Fig. 5(a) that the level of plate vibration reduces due to
the presence of the fluid coupling and also seen the shift of the cut-on frequencies. Fig. 5(a) also shows that the three stiffeners evenly spaced on the base plate reduce the vibration dominantly around the lowest three flexural waves because these waves are most strongly affected by the presence of the stiffeners. In case of the double layered plate, the vibration level of the bottom plate is largely suppressed. In Fig. 5(a), the vibration level of the double layered plate humps around 350 Hz, 1200 Hz and 2500 Hz ranges, in which relatively more waves cut on as shown in Fig. 4(b).

Fig. 5(b) illustrates the sound power radiated from the unstiffened, stiffened and double layered plate with and without fluid coupling. Although the supersonic wavenumber range and vibration level of the base plate shrink much due to the fluid coupling, the level of the sound power is not much reduced in general, except around the first cut-on frequencies. In addition, while the vibration levels are fairly affected by the addition of the stiffeners, the radiated sound power is not much changed by the stiffeners as shown in Fig. 5(b). In case of the double layered plate, the general tendency is similar to the stiffened plate except a level drop around 700–900 Hz, where no waves exist and radiate well as shown in Fig. 4(b).

Average radiation efficiencies calculated from Eq. (6) for unstiffened, stiffened and double plates are shown and compared in Fig. 6. In frequency region below the first cut-on, it is known that the plate radiates sound like a point monopole which has 20dB/decade slope in radiation efficiency.

![Figure 5](image.png)

**Figure 5.** (a) Average mean squared velocities of the base plate, (b) average sound power from the base plate.

![Figure 6](image.png)

**Figure 6.** Comparison of radiation efficiency of 1m width strip plates.
This slope is observed from Fig. 6 below the first cut-on. It can be seen from Fig. 6(a) that the three stiffeners slightly increase the radiation efficiency in whole frequencies. This is mainly caused by the reduced vibration level of the stiffened plate while the sound power remains more or less the same regardless of the stiffeners. The radiation efficiency for double plate shows relatively large increment due to the presence of the upper plate, compare to the stiffeners. However, this result is still predominantly due to the large reduction of vibration level of the bottom plate, rather than the sound power from it. From the results shown in Fig. 5 and Fig. 6, it can be said that the proper radiation efficiency should be used to predict radiated sound power from the fluid coupled plate. If the radiation efficiency of the unstiffened plate is simply used for the prediction of the exterior noise from the stiffened or double layered plate, the less sound power would be obtained.

In the prediction of the radiated noise from ships, subdividing the hull plates based on stiffener spans and using radiation efficiencies of the subdivide plates are often carried out. In order to check the validity of this scheme, a 2 m width plate with evenly spanned three stiffeners of height 0.1 m is examined. For the 2 m width stiffened plate, a point force is applied at $y = 0.85$ m (second bay) and the averaged vibration levels over the whole base plate and each bay are calculated. Then the radiated sound power of the plate is predicted by multiplying the radiation efficiency of the 2 m stiffened plate to the vibration level of the whole plate. Also the sound power is predictable by using the each bay’s vibration levels and the radiation efficiency of 0.5 m width unstiffened plate. Fig. 7(a) shows that radiation efficiencies of the 2m width stiffened plate and 0.5m width unstiffened plate. Fig. 7(a) shows that the radiation efficiency of the 0.5m width plate is higher than that of the 2m width stiffened plate in general. For the force applied, the sound power of the 2m width plate is compared with those predicted by the product of the squared velocity of the plate and radiation efficiency in Fig. 7(b). As shown in Fig. 7(b), the sound power predicted from the four 0.5m width plates is higher than that of the 2m width plate model. From the results in Fig. 7(b), therefore, it can be said that the subdividing a plate based on stiffener span may overestimate the radiated power of the structure.

![Figure 7](image.png)

Figure 7. (a) Radiation efficiencies and (b) radiated sound powers calculated from the 2 m width plate with three stiffeners.

4. Conclusion

In this study, the vibration and sound radiation characteristics of stiffened strip plates coupled with fluids were investigated numerically by using the WFE/BE method. From this study for the fluid coupled plates, it was found that the stiffeners attached to the plate reduce the level of vibra-
tion. The effective frequency range of the stiffeners depends on their locations attached in the plate. On the contrary, it was found that the sound power radiated from the plate into water is not much affected by the presence of the stiffeners. The similar tendency has found from the double layered plate as well. The vibration level of the bottom base plate has decreased much due to the added upper plate but the radiated sound power was not reduced as much as the vibration. From the results of this study, it can be stated that the stiffeners and added plate layer does not significantly reduce the radiated sound power of the fluid coupled plate. However, the radiation efficiency increases because the vibration level is considerably reduced due to the presence of the stiffeners and added layer. Therefore, the proper radiation efficiency should be used to predict radiated sound power from the stiffened or double layered plate coupled with fluid. Finally, it was found that the sound power predicted by subdividing the plate with the stiffener span and integrating each plate’s radiated power may become overestimated than that from the single whole plate.

In the current study, the cavities between the upper and bottom plate were not included. In future work, they also need to be modelled with WBEs then the cavity effect in vibration and sound radiation would be regarded.

REFERENCES