NUMERICAL CONSIDERATION OF CRACKING SOUND GENERATION IN HUMAN JOINT BASED ON BUBBLE DYNAMICS

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As is empirically known, bending a human joint generates sound. This paper theoretically and numerically investigates sound wave (or shock wave) generation and propagation caused by cracking of the human joints (e.g., joints with hard tissues, such as those in fingers, necks, and ankles) on the basis of the classical procedure of bubble dynamics. The Rayleigh-Plesset equation is applied to determine the nonlinear oscillations of bubble that cause sound generation, as observed by Minami et al. (1993). Comparing the numerical solutions of the Rayleigh-Plesset equation for two types of modeling (forced- and free-oscillator assumptions) yields a quantitative difference. Additionally, we develop a physico-mathematical model describing the sound generation caused by cracking a human joint.

1. Introduction

The sound generated by the manipulation of joints is commonly observed in our daily lives. Joints that typically generate an audible cracking sound include those in fingers, knees, and necks. According to previous experimental and theoretical studies, cavitation is formed in the synovial fluid of these joints when a cracking sound is produced\(^1\)\(^2\)\(^3\)\(^4\)\(^5\)\(^6\)\(^7\). When the articular surfaces of a joint are sufficiently separated during an adjustment, an unstable condition arises: The pressure within the cavitation is lower than that in the surrounding fluid, resulting in a net flow of synovial fluid toward the low-pressure region. Because of the release of vibratory energy, the cavitation vibrates.

Understanding the origin of the aforementioned cracking sound is highly important because of its medical and bio-acoustical relevance. Joint manipulation seems to be damaging to the articular capsule or bone\(^8\)\(^9\); therefore, the phenomenon is interesting from a medical viewpoint. In addition to medical interests, the phenomenon is intriguing from the perspective of bio-acoustics. Sound is caused when a bubble in the synovial fluid implodes and generates a pressure wave. The details of this behavior, i.e., the conditions necessary to generate an audible sound, are not fully understood. Because bubble oscillations are nonlinear phenomena, the phenomenon is very interesting from the
perspective of nonlinear dynamics, which may be involved in excitations such as chaos or soliton in synovial fluid or other fluids in the human body. Thus, considerable attention has been directed toward the audible sound caused by the cracking of joints. Many hypotheses have been proposed to explain the generation of this sound; however, our understanding of the mechanism underlying the cracking sound is currently incomplete\textsuperscript{10,11,12}.

This paper presents a theoretical study using a mathematical model to reveal the origin of the cracking sound. To investigate the sound, we used the Rayleigh-Plesset (RP) equation, which is one of the simplest equations describing bubble dynamics. Numerical analysis was performed to solve the RP equation. We discovered that bubble velocity peaks sharply immediately after a bubble is generated. This result indicates that the cracking sound is audible at only one instance, which is consistent with our experience regarding joint manipulation.

2. Physico-Mathematical Model

As reported in the previous work\textsuperscript{13}, the cracking sound is a complex flow phenomenon accompanied by cavitation\textsuperscript{14} and violent oscillations and destruction of many bubbles. We emphasize that our focus is not to make a complete numerical prediction based on strongly nonlinear computational mechanics; rather, we aim to construct a simple and rigorous physico-mathematical model incorporating the most important and fundamental principles of the cracking sound.

We consider a single bubble in incompressible liquid and assume that volumetric oscillations of the bubble (i.e., contraction and expansion) are spherically symmetric, as shown in Fig. 1. The bubble does not appear and disappear, and the compressibility and surface tension of the surrounding liquid are dismissed. The dynamics of the bubble are governed by the well-known RP equation\textsuperscript{15}. Although many models for bubble dynamics have been proposed (e.g., the Keller-Miksis equation, which includes the compressibility of the liquid phase), using a more precise equation would not significantly affect our procedure. The following RP equation was derived from the conservation laws of mass and momentum for the surrounding liquid with the boundary condition given by the velocity of the bubble wall (i.e., the radius $r = R(t)$; $R$ is the bubble radius and $t$ the time):

\begin{equation}
R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 = \frac{1}{\rho} \left[ p_0 \left( \frac{R_0}{R} \right)^3 - p_e(t) - \frac{4\mu}{R} \frac{dR}{dt} \right].
\end{equation}

\textbf{Figure 1.} Single bubble of the radius $R(t)$ in the liquid of the density $\rho$.

Here, $\rho$ is the density of the surrounding liquid, $p_0$ is the hydrostatic pressure, $R_0$ is the bubble radius in the initial state, $\gamma$ is the polytropic exponent ($\gamma = 1$ is used because we consider the isothermal change inside the bubble), $\mu$ is the viscosity of the liquid and is dependent on the temperature, and $p_e(t)$ is the driving pressure. The second term on the left-hand side, $(dR/dt)^2$, represents the second-order nonlinearity. The momentum-conservation equation (or equation of motion) of fluid mechanics\textsuperscript{11} clearly results in the nonlinearity, i.e., the nonlinear dynamics or nonlinear oscillations, in Eq.
Eq. (1) describes a forced oscillator (i.e., simple oscillations of the spring-mass system), and its left- and right-hand sides express the inertia and driving forces, respectively. We apply driving force to the bubble, thus inducing contraction and expansion:

\[ p_\infty(t) = \sin \omega t. \]  

Here,

\[ \omega = \frac{1}{R_0} \sqrt{\frac{3 p_0}{\rho}} \]  

is the eigenfrequency of spherically symmetric linear oscillations of the single bubble.

### 3. Numerical Prediction

We solve Eq. (1) using the fourth-order Runge-Kutta finite difference method with the following initial condition:

\[ R = R_0, \quad v = \frac{dR}{dt}. \]  

Although \( R \) should be set according to the size of the synovial space occupied by the synovial fluid, we do not consider specific tissue in the present paper. \( R_0 \) was then set to 1 \( \mu \)m. As shown in Table 1, the present study uses physical quantities not of synovial fluid but of water, where the temperature was set to the natural body temperature of a human. This was done for simplicity and to enable analogous experiments in future.

**Table 1.** Physical quantities which we use in Eq. (1).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ( \rho )</td>
<td>1.000 [kg/m³]</td>
</tr>
<tr>
<td>Viscosity ( \mu )</td>
<td>0.0008 [Ns/m²]</td>
</tr>
<tr>
<td>Hydrostatic pressure ( p_0 )</td>
<td>101,325 [N/m²]</td>
</tr>
</tbody>
</table>

**3.1 Free oscillator**

First, we consider the case of a free oscillator without any external force. Figures 2(a) and (b) depict temporal variations in bubble-wall velocity and radius for initial bubble radii of 1 \( \mu \)m. A large peak is observed near \( t = 0 \) for velocity, and the corresponding frequency is within the audible range for humans. Because the cracking sound is known to occur only at one instance and this is consistent with our result, oscillation and destruction of bubbles affect the cause of sound genera

![Figure 2](image-url)

**Figure 2.** Temporal evolution of (a) Velocity and (b) Radius of the bubble in the case of \( p_\infty(t) = 0 \).
tion. Temporal variation in bubble radius depicts increasing function that is convex upward.

The source of the cracking sound is synovial fluid near a joint. Therefore, increasing $R$ and the amount of collisions involving a bubble leads to the destruction and extinction of the bubble. We suppose that an increase in $R$, that is, bubble expansion, leads to collisions and the destruction of a bubble (Fig. 3). A previous study\textsuperscript{16} reported that bubbles created in fluid became extinct with time.

### 3.2 Forced oscillator

In real situations, the external force appearing in Eq. (2) periodically drives a bubble. Next, we consider this external force and examine the difference between free and forced oscillators. Figures 3(a) and (b) illustrate the dependencies of bubble velocity and bubble radius, respectively, on external force. Note that the dependence of initial bubble radius on bubble velocity is negligible.

We examine the difference between Fig. 2(a) and Fig. 3(a). In both figures, bubble-wall velocity significantly increases near $t = 0$ and converges to a large peak, subsequently decreasing with time. The most important difference is that in the case of a forced oscillator, oscillation has the form of a wave, as indicated by the sinusoidal driving force in Eq. (2). Although the appearance of multiple peaks in bubble-wall velocity implies multiple occurrences of the cracking sound, we empirically observe only one cracking sound. These results indicate that the variation in bubble-wall velocity due to external forces in real situations is small compared to the present computation.

As is clear from a comparison between Figs. 2(a) and 3(a), the temporal variation in bubble radius [Fig. 3(b)] is qualitatively similar to that in bubble velocity [Fig. 3(a)]. Because the bubble radii in both figures are approximately 9 $\mu$m near $t = 0.01$ ms, the variation in bubble radius due to driving force is negligible. This is because the net value obtained by temporal averaging of the sinusoidal driving force equals zero.

![](image)

**Figure 3.** (a) Velocity and (b) Radius of the bubble, where $p_x(t)$ is given by Eq. (2).
4. Conclusions

We computed the velocities of expansion and contraction of a single spherical gas bubble in water using the classical RP equation. The results are summarized as follows: (i) In the case of a free oscillator, with a bubble radius of 1 μm, a sharp peak in bubble velocity was observed. This is a theoretical validation of the empirical observation that the cracking of a joint produces audible sound at only one instant. (ii) In the case of a forced oscillator, multiple occurrences of a cracking sound were observed.

In a forthcoming study, we will repeat the computation using physical quantities of synovial fluid instead of water and consider the generation and destruction of bubbles. Additionally, the frequency range of cracking sounds should be estimated in order to enable the prediction of audible cracking sounds.

REFERENCES


