ON THE EFFECT OF DRIVING AMPLITUDE, FREQUENCY AND FREQUENCY-AMPLITUDE INTERACTION ON PIEZOELECTRIC GENERATED POWER FOR MFC UNIMORPH

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In this paper, an experimental study on effects of frequency, amplitude and frequency-amplitude interaction of base excitation on vibration piezoelectric energy harvesters is presented. To do so, a unimorph piezoelectric harvester made from macro fiber composite (MFC) piezoelectric layer is tested. A two-factor factorial design with two replications is considered, in which frequency and amplitude of driving vibration are the treatment factors. For changing treatment factors, an appropriate frequency range is considered to include the device fundamental frequency and three excitation levels are considered. In order to investigate the effects of treatment factors on the power output, a linear model is considered. Results show that increasing amplitude of excitation vibration causes a stiffness softening behavior of the piezoelectric oscillatory beam leading to reduction in the harvester natural frequency. In addition, from the frequency-amplitude interaction analysis, output power is much more sensitive to vibration amplitude for driving frequencies near the harvester natural frequency compared to frequencies far away from the harvester natural frequency.

Keywords: Piezoelectric Harvester, Driving Frequency, Vibration Amplitude, Nonlinear Behaviour, Frequency-Amplitude Interaction

1. Introduction

In order to enhance system performance in terms of life time and accessibility for remote systems [1], piezoelectric energy harvesting (PEH) has become an important part of energy harvesting techniques because of its simplicity, easy integration [2] and availability of vibration in any environment [3]. In PEH, frequency and amplitude of vibration source have great effects on output power. Consequently, researchers focused on the study of influential parameters on the output power from PEH [4], [5].

Excitation frequency is the most critical parameter in PEH, as frequency matching to the natural frequency of harvester is essential in order to obtain suitable amount of power in practice. Therefore, two types of harvesters exist, namely linear and nonlinear generators [3]. In linear generators, there are two optimum frequencies for operating a piezoelectric harvester, namely fundamental natural frequency of the device and the anti-resonance frequency [6], depending on electrical and mechanical properties of the device. Roundy et al. [5] investigated the dependency of power output versus driving frequency, added mass, and piezoelectric coupling coefficient based on simulations. In the previous studies, only a one-factor analysis has been carried out that only considered one parameter in each study.
Amplitude of vibration is another important factor affecting generated power. Wei and Jing [7] and Roundy et al. [8] using a general linear and inertial-based generator, Erturk and Inam [9] using a beam-type model, and De Marqui Junior et al. [10] using a two-dimensional finite element model showed that for harmonic base excitation, the power is dependent to the square of external vibration magnitude. On the other hand, in the nonlinear framework, natural frequency of the harvester as a dynamical system can experience a minor change due to high amplitude vibration [11] or due to other parameters such as preload [4]. Evensen [11] showed that by increasing the vibration excitation on a beam, a degree of nonlinearity in fundamental frequency will emerge. This nonlinear effect is much more prominent for the boundary conditions those that are more flexible [11]. In the spite of the nonlinear effect of vibration amplitude on the beam resonance frequency and the fact that piezoelectric harvesters are mostly clamped-free flexible beams, there is no previous studies presented this effect on piezoelectric power output.

PEH is a research area, which has close connection to harvester vibration characteristics and yet there are many vibrational phenomenon that should be addressed. To investigate the interactions of driving frequency and amplitude on piezoelectric generated power, a two-factor factorial design is considered for the experimental test, in which frequency and amplitude of driving vibration are two treatment factors. Then with two replications, a set of runs with different treatment factor levels is carried out. Using an experimental model for the designed tests, main effects of frequency and amplitude as well as frequency-amplitude interaction are investigated.

2. Experimental Setup

Excitation frequency and magnitude can cause important changes in the power generated by piezoelectric materials. To track these changes at different frequencies, a set of experiments are designed to investigate the effect of excitation frequency and amplitude of excitation on piezoelectric harvester.

Unimorph geometry as one the commonest configuration of piezoelectric harvesters was used in this study. Piezoelectric sample comprises of a Macro Fiber Piezoceramic-Composite (MFC M2814 P2) from Smart Materials Corporation [12] bonded to an aluminium substrate with epoxy rapid 332 adhesive with the density of 1.16 g/cm3. Centre shim is a 0.12-mm thickness aluminium shim with elastic modulus of 68.9 GPa and density of 2.7 g/cm3. Piezoelectric harvester was clamped at one end with a clamp box as shown in Figure 1 and then was connected to a VSD 201 shaker for base excitation. VSD 201 shaker is fed by a Kepco AC power generation. National Instruments modules NI 9263 and NI 9215 were connected to a National Instrument Compact data acquisition system (cDAQ) type 9172 for performing signal generation and data recording, respectively. LabVIEWTM 2013 was used as the graphical interface between computer and experimental equipment. Figure 1 shows the experimental setup.

![Experimental Setup and piezoelectric harvester](image1.png)
It is aimed to investigate the dependency of harvested power to excitation magnitude over a range of frequencies containing device natural frequency. Since there are frequency and amplitude of excitation as input factors, a two-factor factorial design should be used. The response is the RMS of output power when the piezoelectric harvester is connected to a 31500-Ω resistance load. Frequency factor is named Factor A and is a continuous parameter rather than a factor with discontinuous levels, hence a random sampling technique will be used. Factor B is the levels of excitation and is a level-based factor. Two replications are considered for the test as well as 40 frequencies with three amplitude levels of 0.1, 0.2, and 0.25 V for shaker input. Therefore, in total, 240 runs will be tested.

3. Results and Discussion

In this section, outcomes of this paper will be presented. It is categorized into three subsections. The first part presents the primarily assessment of experimental data. The next two subsections deal with the investigation of the trend in changing output power with respect to the magnitude and frequency of excitation.

3.1 Primarily Experimental Data

Figure 2 shows the RMS of generated power at different frequencies and with different levels of amplitude excitation, e.g., 0.1, 0.2 and 0.25 V. Power responses for two replications in Fig. 2 show that the experimental runs are met with each other. A well-known fact also is proven here; output power is maximum at the resonance frequency for all three excitation amplitudes. However, by comparing power responses for three different amplitudes, it can be observed that resonance frequency was decreased by increasing excitation amplitude. This behaviour shows that another factor will play a role in piezoelectric power generation.

Figure 2: RMS of power obtained from experiments at different amplitude levels versus frequency

3.2 Nonlinear Behaviour of Unimorph

From vibration theory for a viscous damped single degree of freedom system, oscillatory frequency will be $\omega_d = \omega_n \sqrt{1 - \frac{\zeta_m^2}{m}}$ [13], where $\omega_n = \sqrt{\frac{k}{m}}$ is the undamped natural frequency and $\zeta_m$ is the viscous damping coefficient. The reduction in oscillatory frequency in Fig. 2 can be due to reducing $\omega_n$ or increasing $\zeta_m$. Using the relation between damped and undamped natural frequencies, one can concluded that $\zeta_m$ should increase from 0.01 to 19.1% in order to cause a decrease in $\omega_d$ from 219 Hz at 0.1 V to 214 Hz at 0.2 V provided that $\omega_n$ remain constant. This increase in mechanical damping ratios is not realistic, and hence a reduction in $\omega_n$ is necessary to occur during increase in excitation amplitude for
causing a decrease in $\omega_d$. As the harvester mass remained unchanged during tests, it can be concluded that stiffness of the piezoelectric harvester decreased when it is subjected to higher excitation amplitudes.

If the test with excitation amplitude of 0.1 V is considered as standard and denoted with 0, one can compare the fundamental natural frequency, $\omega_n$, and resonant power output, $P_{\text{max}}$, with respect to the standard state, as shown in Fig. 3. From Fig. 3, it is observed that as excitation magnitude increases, the fundamental frequency of the device nonlinearly decreases, while maximum power increases. The power increase due to vibration amplitude will be thoroughly investigated in the next section. Here, the main focus is the change of piezoelectric resonance frequency due to excitation magnitude. Since piezoelectric resonance frequency decreases with excitation magnitude increase, a softening stiffness behaviour is present in piezoelectric unimorph harvester.

![Figure 3: The effect of excitation amplitude on oscillatory frequency and resonant power](image)

### 3.3 Sensitivity of Interaction Analyses

In this section, it is aimed to perform an appropriate sensitivity investigation on excitation magnitude and frequency. Hence, a statistical model is needed to describe the experimental data for the sensitivity analysis. To do so, a well-known linear model for two-factor factorial design will be used for data treatment. Based on two-factor factorial design, the linear model for these experiments can be expressed as [14],

$$
y_{ijk} = \mu_i + \alpha_i + \beta_j + (\alpha \beta)_{ij} + \epsilon_{ijk}, \quad i = 1, \ldots, 40, \quad j = 1, 2, J = 3, \quad k = 1, 2
$$

(1)

where $i$ associated to frequency level, $j$ to amplitude level and $k$ is the replicates. In Eq. (1), $y_{ijk}$ is the RMS of output power, $\mu$ is the grand mean of RMS power, $\alpha_i$ is the effect of frequency, $\beta_j$ is the effect of excitation amplitude, $(\alpha \beta)_{ij}$ is the interaction effect of frequency-amplitude and $\epsilon_{ijk}$ is the experimental error. It is assumed that the experimental errors are independent and normally distributed with zero mean and standard deviation of $\sigma^2$, e.g. $\epsilon_{ijk} \sim N(0, \sigma^2)$. This assumption later on will be evaluated.

The model in matrix form can be represented as [14],

$$
[Y] = [X][\Psi] + [\epsilon] = [1] \parallel [X_a] \parallel [X_{\beta}] \begin{bmatrix} \mu \\ \Psi_a \\ \Psi_{\beta} \end{bmatrix} + [\epsilon]
$$

(2)

where $[Y]$ is the 12 $\times$ 1 response matrix, $[X]$ is the 12 $\times$ 12 coefficient matrix, $[\Psi]$ is the 12 $\times$ 1 matrix containing grand mean, main effect, and interactions, and $[\epsilon]$ is the 12 $\times$ 1 error matrix. In order to solve Eq. (2) for $[\Psi]$, one can easily state that $[X]'[X]$ should not be singular. As $[X]'[X]$ is singular, a
method should be used in which \( \mu_j \) cell mean is made the standard and consequently \( \Psi \) will be changed to \( \Psi' \), by dropping the first level of factors in the main effect [14]. By this transformation, treatment effect and the error sum of squares (ssE) can be expressed with equations (3) and (4).

\[
\Psi' = (X'[X])^{-1}[X]'[Y]
\]  

(3)

\[
ssE = [Y]'[Y] - \Psi'^{[X]'[Y]}
\]  

(4)

As it was mentioned, it is assumed that the residuals are independent and normally distributed. Figure 4 (a) shows the experimental residuals versus frequency. It can be seen that residuals scattered randomly for different frequencies, emphasizing residuals’ independency. Moreover, as it can be seen from normal Q-Q plot in Fig. 4 (b), experimental data are located close to the fitted line, showing that the residuals can be considered normally distributed figures.

![Constant Leverage: Residuals vs Factor Levels](image)

![Normal Q-Q](image)

(a) (b)

Figure 4: (a) Experimental residuals versus frequency and (b) Norm Q-Q plot

Figure 5 shows residuals of experimental units at different treatment levels. For first experimental unit, frequency, Fig. 5 (a) shows a randomly distributed residuals over frequency range. However, for the second experimental unit, residuals increase with the increase in excitation amplitude. This shows that generated power by piezoelectric harvester is not a linear factor of excitation amplitude. This conclusion is already known from literature, such as Roundy et al. [8].

![Distribution of residuals at different levels of experimental units](image)

(a) (b)

Figure 5: Distribution of residuals at different levels of experimental units

Figure 6 shows the main effects, \( \alpha \) and \( \beta \), in the model presented by Eq. (1), with respect to the marginal average of the other factor. As it can be seen from Fig 6 (a), generated power is maximized at a specific frequency, which is equal to resonant frequency of the harvester with that excitation amplitude. In addition, Fig. 6 (b) shows that increasing the excitation level will lead to dramatic increase in the output power. This conclusion with the analysis of residuals in Fig. 5 (b) proved that the increase in the power with respect to excitation amplitude is exponential rather than linear.
Figure 6. Main effects of treatment factors (a) frequency ($\alpha_i$) and (b) excitation amplitude ($\beta_j$) effects

Figure 7 shows the interaction effects of frequency-amplitude on generated power. Each curve represents output power over different excitation levels with the same excitation frequency. As it was shown in Fig. 6 (a), power is maximum at 214 Hz. In order to observe interaction effects at different frequencies, a set of frequencies ranging from far away from natural frequency, for instance 206, 210, and 222 Hz, to the natural frequency is considered. Figure 7 shows that frequency-amplitude interaction is much more prominent at frequencies close to the natural frequency. For instance, curves assigned to frequencies 213 and 214 Hz having the highest dependency to excitation amplitude while the interaction effect will be lesser for frequencies far away from natural frequency.

Figure 7: Frequency-amplitude interaction ($\alpha_i\beta_j$) on average of RMS power

4. Conclusion

This paper dealt with an experimental investigation of driving amplitude, frequency and frequency-amplitude interaction on piezoelectric output power from a MFC piezoelectric harvester in unimorph geometry. Varying excitation frequency showed that the output power is maximum while the device is vibrating at its natural frequency. On the other hand, increasing amplitude of vibration will lead to the exponential increase of power output. By considering frequency-amplitude interaction, two main conclusions were made. Firstly, piezoelectric harvester in unimorph geometry represented a nonlinear stiffness softening behaviour in such a way that resonance frequency will decrease as excitation amplitude
increases. Furthermore, the sensitivity of output power to driving amplitude is different for different driving frequencies in such a way that output power is much more sensitive to the driving amplitude at frequencies near resonance frequency.

REFERENCES


