PARAMETRIC LOUDSPEAKER: FROM THEORY TO APPLICATIONS

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A review of the current state of parametric loudspeaker from the aspects of modelling, measurement, signal processing, implementation and applications is presented in this paper. A variety of well-established and emerging numerical methods to predict the propagation of finite amplitude sound beams are discussed and compared. The accurate measurement of the difference-frequency sound is another research interest related to parametric loudspeaker. The measurement methods using an acoustic filter are outlined, and the approaches by taking advantage of condenser microphone sensitivity characteristics or phase cancelling technique are also shown. The signal processing algorithm to reduce the harmful effects of the harmonic distortion also has constantly attracted the attention of researchers. The processing method based on Berktay’s solution together with the new approaches using psychoacoustic phenomenon or Volterra filter modelling are given. Furthermore, the detailed implementation of parametric loudspeaker is summarized and its potential application is listed. This paper will focus on the recent advance in the parametric loudspeaker field from theoretical perspective as well as engineering perspective to promote the theoretical and practical development of the parametric loudspeaker.

1. Introduction

By utilizing the nonlinearity of the acoustic waves in air, parametric loudspeaker can be used to create sound source with higher directivity at a significantly smaller aperture compared to conventional loudspeakers. The parametric loudspeaker has been used in underwater sonar applications for decade, here it is generally referred as the parametric array deployed in air 1-4. The basic idea of the parametric array was originally conceived by Westervelt about 50 years ago based on the scattering of sound by sound 5-6. The original work by Westervelt 7 addressed the generation of primary waves, namely difference-frequency signal from two high-frequency collimated beams. The directivity of the parametric array is attributed to an end-fire array of virtual sources, which are the by-products of the nonlinear interaction between the primary waves propagating in the medium. A more complete explanation of the parametric array was given by Berktay 8 and his analysis was not limited to
two primary waves. Assumed that the primary wave is in the form of \( p_1 = P_0 E(t) \sin \omega t \) with concept of the modulation envelope, a farfield solution can be obtained

\[
p_2 \propto \frac{P_0^2 \partial^2 E^2(t)}{\partial t^2},
\]

where \( t \) is the time, \( E(t) \) is the envelope function, \( P_0 \) is the initial sound pressure level (SPL) of the carrier and \( p_2 \) is the demodulated signal. The Berktay’s farfield solution, which defines the demodulated secondary waveform generated along the axis direction of the beam, is proportional to the second-time derivative of the square of the envelope. Parametric loudspeakers have attracted intensive interest since the theoretical study of underwater application was initiated in 1950s. Since the discovery of parametric array by Westervelt, hundreds of papers and technical reports on this topic have been published from theoretical and experimental points of view. However, almost all the papers before 1975 focused on exploratory examinations and applications in underwater. An experimental investigation, conducted by Bennett and Blackstock, has proved that an audio sound with high directivity can be generated only in the target region without affecting other areas when two primary waves of different frequencies interact in air. Yoneyama et al. have modulated ultrasonic waves with audio signals using double sideband amplitude modulation (DSBAM) method, which was regarded as the beginning of the practical application of parametric loudspeakers in audio products.

The parametric loudspeaker has attracted considerable interest in the acoustic engineering field, and a substantial number of academic research papers have been published about the basic theory and key technologies. Some important advances have been achieved both in the engineering and research fields of the parametric loudspeaker. In this paper, we review some of these key research works, and report on new advances in the parametric loudspeaker, including numerical modelling for nonlinear acoustics, difference-frequency sound measurement, signal processing techniques, implementation issues and applications.

2. Numerical modeling for nonlinear acoustics

It is well known that the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation can accurately describe the propagation of finite amplitude sound beams by combining the effects of diffraction, absorption, and nonlinearity under a parabolic approximation. The KZK equation is widely used for theoretical prediction of the nonlinear field of a directive sound beam, and the validity of the KZK equation for the parametric array has been verified by many researchers. This model equation is described as:

\[
\frac{\partial^2 p}{\partial z \partial t'} = \frac{c_0^2}{2} \nabla^2 p + \frac{\delta}{2c_0^3} \frac{\partial^3 p}{\partial t'^3} + \frac{\beta}{2\rho_0 c_0^3} \frac{\partial^2 p^2}{\partial t'^2},
\]

where \( p \) is the sound pressure, \( z \) is the coordinate along the axis of beams, \( c_0 \) is the sound speed, \( \rho_0 \) is the medium density, \( \delta \) is the sound diffusivity related to thermoviscous absorption, and \( \beta \) is the nonlinearity coefficient. Moreover, \( \nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2 \) is a Laplacian that operates in the \( x-y \) plane perpendicular to the axis of the beam (\( z \) axis), and \( t' = t - z / c_0 \) is the retarded time. The three terms on the right-hand side of Eq. (2) account for diffraction, nonlinearity, and thermoviscous absorption respectively. Khokhlov and Zabolotskaya derived the lossless form of the KZK equation, i.e., KZ equation, and Kuznetsov considered the effect of thermoviscous absorption. There is still no explicit analytical solution to the complete KZK equation. Instead, numerical methods have been developed, including frequency-domain algorithm, time-domain algorithm, and combined time-frequency domain algorithm, and a review of these methods can be found in the reference. Bakhvalov and his coworkers have carried out the numerical study about the KZK equation in a series of articles. They compute the effects of absorption and
diffraction in the frequency domain, and effect of nonlinearity in the time domain. The first spectral algorithm was Bergen Code which was developed by Aanonsen et al. to calculate the near field of finite-amplitude sound beams\(^9\). The core of Bergen Code is to expand the sound signals into the Fourier series and solve every frequency component individually. Subsequently, the spectral algorithm was expended to calculate the far field\(^{16}\), bi-frequency source\(^{17}\), focusing beams\(^{18}\) and non-axisymmetric source\(^{19}\). The time-domain algorithm was developed by Lee and Hamilton\(^{20,21}\), known as the Texas Code, which avoids the truncation of the Fourier series in the spectral algorithm. The advantages of using the time-domain KZK algorithm are 1) its ability to compute arbitrary time waveform, and 2) computational efficiency\(^{27}\) as compared to other previous methods. However, the original Texas Code just aimed at the thermoviscous medium and did not consider the relaxation effect, which was completed by Cleveland et al.\(^{28}\). Liu et al. developed a three-dimensional time-domain finite difference algorithm for solving an augmented KZK equation\(^{29}\) based on Transformed beam equation (TBE), where the relaxation effect is also included. Figure 1 illustrates the sound fields of the difference frequency with the same parameters in the reference\(^{19}\), where Figure 1(a) is the axial fields and Figure 1(b) corresponds to the beam patterns at \(z = 6\) m in the \(y\) direction. The results calculated by Kamakura et al.\(^{19}\) are given in Figure 1 for comparison. One can see that our results agree well with those obtained by Kamakura et al.\(^{19}\). The maximum deviation is less than 1 dB and exists in the situation of high sound pressure level (\(p_0 = 128\) dB).

![Figure 1](image)

**Figure 1.** The sound fields of the difference frequency calculated by using the present 3D time-domain code: (a) the axial field; (b) \(y\)-directional beam patterns at \(z=6\) m.

In the case of weak nonlinearity, the quasilinear solutions to the KZK equation have been obtained for the fundamental and second-order sound fields using the successive approximation method\(^9\), which are based on the assumption that the second order solutions are small corrections to the corresponding linear theory. It ignores all the harmonic components larger than second order. Under the quasilinear approximation, the sound fields are reduced to the sum of the first order and second order fields which can be expressed as the integral. This is a heavy numerical task. To solve this problem, the Gaussian beam expansion technique has been widely used in modelling the fundamental and the second-order wave field for its high computational efficiency\(^{20-38}\). The principle of this technique is to decompose the source distribution function into the superposition of a small number of Gaussian functions. An advantage of using the Gaussian-beam expansion technique is that an analytical description of the fundamental sound field propagation can be obtained, while a simplified algorithm for the second-order sound fields can be developed, leading to fast computation. Ding\(^{30}\) used this approach to simulate elliptical and rectangular transducers. Yang et al.\(^{35-36}\) obtained the expansion coefficients for rectangular and elliptical planar piston transducers in a two-dimensional spatial Fourier transform \((k\)-space\) domain. An overview of the Gaussian-beam expansion technique has been given in the reference\(^{39}\). The key challenge here lies in the determination of the coefficients of Gaussian functions. In the previous studies, Wen and Breazeale developed the spatial domain method\(^{37,38}\) and Sha et al. adopted the \(k\)-space domain method. However, the meth-
ods which use nonlinear optimization algorithms are complicated and time-consuming \(^{35-36}\). This brings forth the need to find a simple algorithm to calculate the Gaussian beam expansion coefficients. Ding and Zhang proposed a linear scheme in spatial domain \(^{33}\). The obvious advantage of this scheme is the simple determination of the Gaussian beam expansion coefficients by solving a set of linear equations. However, since half of the predetermined coefficients are real, the practicality of this scheme is limited. Liu et al. developed a fast algorithm to determine the Gaussian function coefficients for a more accurate approximation with two-stage procedures \(^{40}\). Firstly, two real coefficients are estimated by a simple search approach, and then the least mean square (LMS) algorithm is adopted for determining the optimal expansion coefficients. Compared to the previous methods, the new method has a better accuracy in far field. To exhibit the good performance of this algorithm, here a simple example of the diffraction field of a circular source with 15 term Gaussian beam expansion coefficients is given. The source distribution function is given in Figure 2(a). It can be seen that the current result gives a better fit of the original source function than that given by Wen and Breazeale’s results. Figure 2(b), 2(c), and 2(d) illustrate the axial field, the relative error of the axial far field, and the profile distribution at distances of 0.25\(z_0\) (\(z_0\) is Fresnel distance), respectively. It can be seen that our results agree well with the Rayleigh integral (with \(ka = 107.8\)) except in the very near field. Compared with Wen and Breazeale’s results, our results are better in far field (the relative error in Figure 2(c) is 0.05% versus 1%).

![Figure 2](image.png)

**Figure 2.** Comparison of the wave field of the uniform piston source calculated by using the coefficients of the present method, those of previous method, and Rayleigh integral: (a) the source distribution function; (b) the axial field; (c) the relative error of the axial far field; (d) \(z = 0.25z_0\).

Since the KZK equation is derived from a full nonlinear wave equation known as the Westervelt equation under the parabolic approximation, it introduces errors at field points that are far away from the acoustical axis (\(>20^\circ\)) at locations close to the source \(^9\). Several approaches have been proposed to overcome such limitations, most of which are based on the Westervelt equation. Huijssen et al. obtained the axial and lateral beam profiles of harmonic components by applying the
explicit finite-difference time-domain (FDTD) scheme to the Westervelt equation. Based on the Neumann iterative solution, Huijssen and Verweij proposed a novel method to solve the lossless Westervelt equation, where the nonlinear term is treated as a nonlinear contract source. Kamakura et al. solved the Westervelt equation by introducing the split-step Padé approach and extended its applicability limit of propagation angle to ±40°, which is twice as wide as that of the KZK equation. Recently, Angular Spectrum Approach (ASA) has been widely used in ultrasound simulations due to its rapid computation time. ASA was firstly introduced in nonlinear acoustics by Alais and Hennion to analyze sum- and difference frequency generation in the parametric array. Landsberger and Hamilton studied the second harmonic generation of sound beams in immersed elastic solids. A simulation package named as Abersim has been provided by Frijlink et al. to solve the directional Westervelt equations in Cartesian 2D and 3D grids. Although it adopted ASA in the calculation of diffraction effect, its computation time will increase dramatically if the nonlinear term is taken into account. Du and Jensen adopted the ASA for solving the nonlinear wave equation for the second harmonic, using the source generated by Field II based on the Westervelt equation. It computes much faster than Abersim, since the ASA is an analytical solution and the simulated point can be computed in one step. Based on the Gaussian beam expansion technique in the quasilinear approximation, Cervenka and Bednarik also obtained a formula to calculate the difference-frequency wave by a baffled axisymmetric piston with the method of successive approximations.

3. Difference-frequency sound measurement

The accurate measurement of the difference-frequency sound in the parametric loudspeaker is vital for understanding the principle of the parametric loudspeaker and also the practical design in audio engineering. However, spurious sound will be generated as a result of nonlinearity at the receiving system caused by the existence of the finite-amplitude ultrasonic waves, which are referred as the primary waves. The measured signal will be a combination signal including both the difference-frequency sound from the parametric loudspeaker effect and the spurious signal. Hence, accurately measuring the difference-frequency sound will become a problem, and the situation is particularly worse in the nearfield and near the beam axis.

A few methods have been proposed to measure the difference-frequency sound of parametric loudspeaker in the literature by adopting an acoustic filter mounted in front of the receiving transducer to suppress the levels of the primary waves. In the first experimental verification on the parametric loudspeaker in air, Bennett and Blackstock adopted a dome-shaped filter made of a 2.5 mm thick clear plastic material, which reduced the SPL of the primary waves by about 20 dB at a cost of a 3.5 dB SPL reduction for the difference-frequency signal. The effect of nonlinearity within the receiving system was also found in water by Garrett et al. and it was proportional to the product of the primary wave at the measuring point, which was supported by the experimental results for both the axial propagation and the beam patterns. When Lucas et al. investigated the focused parametric array in water, they implemented a sheet of rubber (1.3 cm thick) as an acoustic filter, which provided an attenuation of 16 dB for the primary wave with an undesired 2.5 dB reduction for the difference-frequency signal. Kamakura et al. used a sheet of air-pad in their experiments and it reduced the primary wave at 40 kHz by over 20 dB. Havelock and Brammer installed a planar sound absorber made of four layers of 3.18 mm felt in their experiments and achieved an SPL reduction at 30 kHz by over 20 dB with an unwanted attenuation for the audio frequencies by 3-5 dB. Toda presented a complex acoustic filter having four polymer layers, which was found to have a 30-40 dB attenuation in the range of 30-40 kHz with an unwanted ±5 dB deviation in the range of 10 Hz-10 kHz. Humphrey et al. adopted a 30 mm thick polyurethane panel as the acoustic filter when they evaluated the underwater acoustic performance of panels by using a parametric array. Wygant et al. adopted a thin sheet of Saran film as the acoustic filter,
and they found that the experimental results with the acoustic filter match well with the simulation results. A 16-mm-diameter aluminum plate was developed as the acoustic filter by Ye et al. and a 15 dB attenuation of the primary wave with an undesired ±5 dB deviation in the audible range were found by using the filter. The frequency response of the parametric loudspeaker was measured without and with the acoustic filter as shown in Figure 3(a). It was clearly confirmed that the spurious signal was independent of the difference frequency, in the case of without the acoustic filter. The audible sound SPL generated by the parametric loudspeaker is proportional to \( f_n \), where \( n \approx 1.6 \). In order to further distinguish the audible sound generated by the parametric loudspeaker from the spurious signal of the transducer, the axial SPLs of the difference frequency sound was measured and shown in Figure 3(b). The spurious signal including the transducer nonlinearity and radiation pressure was modeled by \( p_m = K p_a p_b \), where \( p_a \) and \( p_b \) were the peak amplitudes of the primary waves at the transducer location, respectively. The constant number \( K = 2.8 \times 10^{-9} \text{ Pa}^{-1} \) was determined by curve fitting from the measurement results.

As can be seen from the above literature, using these proposed acoustic filters, the primary waves at the receiving system were definitely suppressed and so did the spurious sound; however, the measured difference-frequency sound was less affected.

![Figure 3. (a)Frequency response of the parametric loudspeaker (b) Axial SPLs of the parametric loudspeaker.](image)

Recently, two alternative methods instead of the acoustic filter in the measurement setup are also presented. Ju et al. proposed a technique by taking advantage of the sensitivity characteristics of the condenser microphone to reduce the levels of the primary waves with a large grazing angle. But it will also affect the sensitivity in the audible range, especially when the signal’s frequency is higher than 10 kHz. Based on the phase-cancellation and the Gaussian beam expansion technique, Ji et al. proposed an alternative method for measuring the difference-frequency sound accurately without using any traditional acoustic filter. But it is only suitable for the axial measurement.

4. **Signal Preprocessing Techniques**

4.1 **Processing method based on Berktay’s solution**

Two major disadvantages of parametric loudspeakers have been noticed from practical applications, as well as from the Berktay’s solution: 1) the square operation of the envelope function inevitably leads to the generation of harmonic distortions; 2) the second derivative operation behaves as a high-pass filter, which yields a poor frequency response at low frequency range, generally below 500 Hz.
By far, amplitude modulation is the dominant modulation technique used in studies and applications of parametric loudspeakers. Though the original DSBAM requires fewer computational resources, the harmonic distortion existing in the secondary wave greatly degrades the sound quality, which can be observed from both the numerical simulations and the nonlinear Volterra modeling results. During the past three decades, various preprocessing approaches in the modulation stage have been proposed to alleviate the harmonic distortion. One solution introduced by Kamakura et al. 62 is to use single sideband amplitude modulation (SSBAM), which transfers the energy of two sidebands in DSBAM to one sideband, and thus lowers the bandwidth requirement of the ultrasonic transducer. As shown in Figure 4, SSBAM is essentially a type of quadrature modulation using two orthogonal carriers as $\cos(\omega t)$ and $\sin(\omega t)$. The Hilbert filter whereby the modulating signal $g(t)$ can be converted to its orthogonal counterpart is easy to implement using digital filter design tools. Moreover, the carrier amplitude can be adjusted based on the envelope of the modulating signal in practical applications. Thus, the total power consumption and the load of ultrasonic transducers could be greatly reduced 63.

![Figure 4. Generation of SSBAM signal.](image)

Subsequently, Kite et al. 64 predistorted the input modulating signal with a square-root operation to the envelope of DSBAM signal to compensate for the distortion brought about by the square operation in the Berktay’s solution. This approach was later validated by Pompei 65 who devised a practical parametric loudspeaker with low distortion. In Figure 5, it should be noted that the square-root amplitude modulation (SRAM) produces a series of frequency components in the modulated signal due to the square-root operation. However, experiments have revealed that SRAM technique is effective to reduce the distortion to an acceptable level around 5% in practical use with certain types of ultrasonic transducers 65.

![Figure 5. Generation of SRAM signal.](image)

Another preprocessing technique involving quadrature modulation is modified amplitude modulation (MAM), which was proposed by Tan. et al. 66 to offer a flexible bandwidth variety for a wide range of ultrasonic transducers. From Figure 6, we can note that one branch modulation is the same as DSBAM, while in the other branch the orthogonal carrier is modulated by a predistorted term $\sqrt{1-m^2g^2(t)}$ to compensate for the harmonics generated in DSBAM. The main advantage of MAM is that $\sqrt{1-m^2g^2(t)}$ can be substituted by its Taylor series of different orders to adapt to the bandwidth of actual ultrasonic transducer. Performance analysis has been carried out in the reference 67, simulation results showed that MAM could achieve a similar THD level compared to SSBAM and SRAM.
The above-mentioned preprocessing techniques mainly focus on solving the harmonic distortion occurred in parametric loudspeaker systems, designers are therefore able to select the proper preprocessing technique according to specific system requirements, such as analog or digital implementations, available computational resource, bandwidth limitation of ultrasonic emitter, etc. To solve other limitation problems of parametric loudspeakers, researchers in recent years have concentrated their attention on enhancing the sound quality in low frequency range. In 2012, Shi et al. introduced the “missing fundamental” concept widely used in virtual bass enhancement for conventional loudspeakers to improve the bass quality for parametric loudspeakers. As shown in Figure 7, this psychoacoustical preprocessing technique splits the audio input with a low-pass and a high-pass filter at a cut-off frequency of 500 Hz. The high-passed audio content is treated using the general preprocessing technique, while the low-passed bass is manipulated by function $F(x)$, which is served as the harmonic generator for parametric loudspeakers. Subjective tests show that this preprocessing method could promote the bass quantity, and maintain a relatively good sound perception simultaneously.

Another method to improve the sound quality and equalize the frequency response of parametric loudspeakers was presented in the reference by combining DSBAM and SSBAM techniques. In this work, the audio input signal is divided into three frequency ranges based on the characteristics of one commercial parametric loudspeaker. DSBAM is used to retain the sound energy in the low frequency range below 2 kHz, while SSBAM deals with the 2-6 kHz components using lower sideband and 6-10 kHz components using upper sideband, respectively. In addition, the modulated signal is weighted to achieve a flat power spectrum of the audible sound in the output terminal. Subjective tests demonstrate that this combined modulation method outperforms the single use of DSBAM or SSBAM in terms of sound loudness and quality, as well as the harmonic distortion.

### 4.2 Processing method based on Volterra series

Those Berktay-based preprocessing methods mentioned above cannot reduce the harmonic distortions effectively when the levels of the primary waves are higher than a certain level due to the limitation of the Berktay’s farfield solution. The nonlinear mechanism in parametric loudspeakers can also be modelled by certain types of filters with nonlinear structures. One versatile approach for characterizing the nonlinear process is to utilize the Volterra series with a straightforward filter structure. In 1995, the nonlinear process in horns and ducts was investigated by Klip-
pel using a finite Volterra series expanded to the $n$th-order kernel. Subsequently, a simplified Volterra filter with one-dimension coefficient vector was proposed by Farina et al. in 2001, and has been successfully applied to nonlinearity modelling of an audio limiter, tube preamps and musical instruments.

As the Volterra series is advantageous in nonlinear system identification, and the nonlinear distortion of the parametric loudspeaker system is mainly attributed to the second harmonic element, an adaptive Volterra filter with the 1st- and 2nd-order kernels in a cascaded structure has been adopted by Ji et al. to model the nonlinearity of parametric loudspeaker systems. The output of the parametric loudspeaker system $y(n)$ can be given as

$$y(n) = y_1(n) + y_2(n)$$

$$= \sum_{m_1=0}^{N_1-1} h_1(m_1)x(n-m_1) + \sum_{m_2=0}^{N_2-1} \sum_{m_1=0}^{N_1-1} h_2(m_1,m_2)x(n-m_1)x(n-m_2)$$

$$= H_1[x(n)] + H_2[x(n)],$$

where $x(n)$ is the input modulating signal, $h_1(m_1)$ and $h_2(m_1,m_2)$ are the 1st- and 2nd-order kernels, respectively. $H_1[\cdot]$ and $H_2[\cdot]$ represent the 1st-order and 2nd-order kernel operators accordingly.

For theoretical analysis, Ji et al. resort to the KZK mathematical model to calculate the secondary sound as the desired signal assuming that the ultrasonic emitter has a flat magnitude response and zero phase shift. The input signal, which is a uniformly distributed random white noise signal, modulates a 40-kHz carrier using DSBAM with $m = 0.8$ to form the primary waves for investigating the quadratic nonlinearity presented in the secondary sound. Figure 8 illustrates the steady state coefficients of the 1st- and 2nd-order kernels after $3.2 \times 10^4$ iterations using the NLMS adaptive algorithm, respectively. It can be observed that the 1st-order kernel approximates the impulse response of a distortionless transmission for linear output, while the 2nd-order kernel indicates that the quadratic nonlinearity is nearly memoryless with a dominant coefficient locating at $h_2(0,0)$.

![Figure 8](image_url)

**Figure 8.** Coefficients of the (a) 1st-order and (b) 2nd-order Volterra kernels.

Conventional Volterra filters have been widely and efficiently applied to nonlinearity modelling of a large class of nonlinear systems. However, due to the complexity of higher order kernels, Volterra series have been seldom implemented in practical parametric loudspeaker systems. One important challenge encountered in real-time system is the requirement of fast system modelling and inverse filter design for predistorting the input signal. To address the issue of high computational complexity, a simplified Volterra filter with one-dimension kernel has been proposed and studied in the reference. Generally, the one-dimension Volterra series for characterizing a discrete nonlinear system can be described as
where $h_p(m)$ is defined as the $p$th-order one-dimension Volterra filter kernel, and $N_p$ is the kernel length. For parametric loudspeaker system modelling, Equation (4) is truncated up to the 2nd-order kernel with $p = 2$.

To obtain the values of the 1st- and 2nd-order one-dimension kernel, an exponential swept-sine signal $x(n) = A \sin \left( \omega_0 T / \ln\left( \frac{\omega_2}{\omega_1} \right) e^{\frac{n \ln(\omega_2)}{N} - 1} \right)$ is used as the input signal, where $A$ is the amplitude, $\omega_1$ and $\omega_2$ are start frequency and stop frequency, respectively, $T$ is the signal length, and $N$ is the sample number. The values of $\omega_1$ and $\omega_2$ should be carefully selected to meet the demand of the dynamic behaviour of the nonlinear system within the sweeping range, and the value of $T$ must be long enough for an acceptable signal-to-noise ratio (SNR).

For a parametric loudspeaker system excited by the exponential swept-sine signal, the system linear and quadratic impulse responses, namely $\tilde{h}_1(n)$ and $\tilde{h}_2(n)$ can be obtained by convolving $y(n)$ with the inverse of $x(n)$, $h_1(n)$ and $h_2(n)$ are calculated using the following formula as

$$
\begin{cases}
    h_1(n) = \tilde{h}_1(n) \\
    h_2(n) = -2\tilde{h}_2(n) / A
\end{cases}
$$

The modelling performances of the conventional Volterra filter and the simplified one-dimension Volterra filter have been evaluated by measurements. Seven input monotones from 0.5 kHz to 3.5 kHz with an increment of 0.5 kHz are passed to an actual parametric loudspeaker system and the two Volterra models, respectively. The performance index is defined as the relative error $E_i$, which is given as

$$
E_i = \left| \frac{H_i[x] - Y_i}{Y_i} \right| \times 100\%, \quad i = 1, 2
$$

where $i$ represents the harmonic order, $Y_i$ is the $i$th harmonic existing in the output of the actual parametric loudspeaker, and $H_i$ is the output of the $i$th-order Volterra kernel. Figure 9 shows the curves of relative error $E_i$ using DSBAM for the conventional Volterra filter and the one-dimension Volterra filter. It can be noted that the modelling errors for both the 1st- and 2nd-order kernels of the conventional Volterra filter could be controlled around 5%, which indicates that the Volterra model is effective to predict the output sound intensity under the same experimental conditions. For the one-dimension Volterra kernels, the linear output could exhibit the similar performance, while the relative errors of the quadratic output using the simplified 2nd-order kernel increase to around 10% due to the approximation error.
5. Implementation issues

In this section, the implementation issues about developing the parametric loudspeakers in analog and digital circuits are summarized. In general, the parametric loudspeaker system consists of three main parts, namely signal processing, amplifier, and ultrasonic emitter, as shown in Figure 10. Initially, parametric loudspeakers were based on analog circuits, especially the parts of signal processing and amplifier shown in Figure 11. The signal processing is divided into two steps. Firstly, the input signal is fed by the peak level detector, which can be designed with a voltage comparator (LM111). The purpose of the peak detector is to avoid the overmodulation and get a positive envelop signal exported from adder that is simply achieved by using an operational amplifier. Afterwards, the envelop signal is preprocessed by square-root circuit implemented by a four-quadrant analog multiplier (AD734). This preprocessing is employed to reduce the distortion produced during the nonlinear interaction in the air propagation, which have been explained in Section 3. Finally, the preprocessed signal is modulated by the carrier signal through the multiplier. The carrier signal is produced by MAX038, which is a high-frequency, precision waveform generator. Although the analog development of a parametric loudspeaker has the advantages of simple structure and low cost, it is lack of stability and flexibility. Furthermore, it is not suitable to implement complicated functions and operations, such as Hilbert filter in the SSBAM scheme and dynamic range control. These unfavourable factors have limited the development of high performance parametric loudspeaker.
Compared with analog circuit, digital circuit is more flexibility in complicated algorithms without incurring extra cost, size, and power consumption. With the advantage of flexible configuration and high performance, Field Programmable Gate Array (FPGA) is a good choice to implement the parametric loudspeaker. The Contemporary FPGA has large resources of logic gates and RAM blocks to implement complex digital computations. FPGA is able to perform parallel operations, as well as serial operations. The operation is chosen depending on the complexity of the algorithm and resource size of FPGA.

The generic block diagram for implementing digital preprocessing and modulation in the parametric loudspeaker is shown in Figure 12. The digital processing is divided into four steps: dynamic range control (DRC) processor, pre-distortion processing, automatic carrier level controller, and amplitude modulator. The DRC processor consists of a compressor and a limiter. The compressor amplifies quiet sounds or reduces the volume of loud sounds by narrowing an audio signal’s dynamic range. The limiter is employed to prevent the sound from clipping that will bring perceivable distortion of the difference frequency sound. The pre-distortion processing has been previously discussed in Section 3, which reduces the amount of harmonic distortion and intermodulation distortion. In the third step, the sinusoidal oscillator generates a carrier signal, and automatic carrier level control produces a dynamical gain. The genetic implementation ways of sinusoidal oscillator include recursive algorithm, a look-up table and direct digital frequency synthesis (DDS). The dynamic carrier signal, which significantly reduces the power consumption, is obtained by multiplying the carrier signal with dynamical gain. In the final step, the input signal is modulated by the dynamic carrier signal.
Figure 12. Digital processing of parametric loudspeaker.

Figure 13. Interconnection between FPGA board and interface board.

Figure 13 shows the connecting diagram between the interface board and the FPGA chip board. The FPGA chip performs the digital signal processing. The interface board consists of an audio codec, voltage regulators and low-pass filters. The audio codec provides a two-channel 16-bit 96 kHz analog-to-digital converters (ADCs) and 192 kHz digital-to-analog converters (DACs) with single-ended analog input/output. As the resonant frequency of the most ultrasonic emitters used for parametric loudspeaker is between 30 kHz and 50 kHz, the bandwidth of 192 kHz DAC is wide enough. In order to reduce the harmonic distortion of analog signals from DAC, low-pass filters are essential. In addition, Voltage regulators produce 3.3 V and 5 V power supply.

In order to create parametric array effect in air, the modulated signal must be power amplified before exporting to the ultrasound emitter. Commonly, the power amplifiers can be divided into types of Class-B, Class-C, and Class-D. The last one is widely used in the parametric loudspeaker due to its high efficiency and small size. Figure 14 gives a scheme of Class-D amplifier used in the parametric loudspeakers, in which the pulse modulator can be implemented by the ways of analog circuit and digital function. Compared to analog circuit, pulse train can be directly converted from the digital signal by digital function without using the DAC. In the term of pulse modulation, there are two common techniques: pulse density modulation (PDM) and pulse width modulation (PWM). PWM generates variable duty pulses, the widths of which are proportional to the input amplitude; PDM generates variable density pulses, the density of which corresponds to the input signal’s amplitude. PDM generates less harmonic distortion than PWM, but has lower efficiency. In the Class-D output stage block, the PWM signals are amplified by power MOSFET transistors to drive the ultrasound transducer. The level of amplified PWM signal reaches the supply voltage. Due to
the presence of dead time during the switching MOSFET transistor, the distortion and noise are brought into the signal. In order to significantly alleviate the problem, an error correction technique is required. Finally, an EMI filter, which should be designed elaborately to remove the high frequency products and maximize the power transfer, is needed before driving the transducers.

![Block diagram of Class-D amplifier for parametric loudspeaker.](image)

**Figure 14.** Block diagram of Class-D amplifier for parametric loudspeaker.

The steerable parametric loudspeaker consists of several sub parametric loudspeakers, each beam of which is delayed by specific time. The steering angle is determined by the value of delay. Figure 15 shows the system structure of the steerable parametric loudspeaker. The digital processing algorithm is basically described in Figure 12, whereas the realization approaches of delay are various. Olszewski et al. realized the system of steerable parametric loudspeaker with hybrid of mechanical tilting and electronic time delays. The whole system includes four sub-arrays, which are controlled by separate stepper motors. This method does not bring in grating lobes, but results in the complexity and bulk of the system. Yang et al. achieved the constant beamwidth beamforming of the steerable parametric loudspeaker using the Chebyshev weighting function and confirmed in the simulation experiments. Gan et al. proposed an algorithm that the carrier and sideband frequencies were separately delayed. Owing to the main lobe of sideband frequency’s directivity is almost flat, the difference frequency directivity’s steering angle and main lobe beamwidth were inherited from the carrier frequency’s directivity. By combining digital loudspeaker technique, Takeoka et al. individually modulated and delayed every element signal of the steerable parametric loudspeaker to emit multiple independent audio beams. Also, Wu et al. used fractional delay algorithm to steer the beam of parametric loudspeaker to any arbitrary angle. Since the spatial sampling theorems are not satisfied, the difference frequency beam presents grating lobes. Pompei et al. presented a method that utilized oversized array transducers carefully arranged to eliminate grating lobes. Shi et al. proposed a Gaussian source array in place of the ultrasonic transducer array to prove the feasibility of devising a digital beamsteerer for the parametric loudspeaker. According to Gaussian source array, the directivity characteristics of linear ultrasonic transducer array were analyzed. Using the delay and Chebyshev weighting, the grating lobe of difference frequency was suppressed. Lee et al. proposed a way to versatile beamforming, using the complex weights in place of time delays. In the term of beamsteering simulation, Shi et al proposed three modified product directivity models to predict the locations and amplitudes of the mainlobe and grating lobes. Compared with the original product directivity model, the models proposed by Shi promoted the selected sidelobe predictions by about 10 dB.
6. Applications

Parametric loudspeaker is capable of creating directional sound beam using an ultrasonic emitter, which has been used in personal entertainment system, communication, and messaging system. In this section, some recent applications are listed which show good potential in exploiting unique features of parametric loudspeaker.

Traditionally, omni-directional loudspeakers are used as control sources to suppress the noise in the targeted area in the active noise control (ANC) system. However, the sound pressure level (SPL) of the noise at locations away from the target area may actually increase. Brooks et al. investigated the feasibility of using a parametric loudspeaker as a control source in the ANC system. Experimental results showed that the ANC system using parametric loudspeaker could reduce the SPLs at the control points without increasing noise level in other areas. Kidner et al. proposed a method of combining the parametric loudspeaker and virtual sensing techniques to create localized zones of quiet. Tanaka and Tanaka presented a novel idea of using a steerable parametric loudspeaker based upon phased array theory with an optimal control law to track a moving target point without rotating the parametric loudspeaker mechanically. Furthermore, the same research group proposed a novel method to use the reflected sound wave, which was produced by the parametric focusing loudspeaker on the target source, to collocate both the primary source and the control source. Therefore, global noise control can be achieved because the distance between the two sources became theoretically nullified. In another work, Komatsuzaki and Iwata compared the interfered sound field of using the parametric loudspeaker with omni-directional loudspeaker as control sources to achieve local noise control. Three different constructions of the primary source and the control source were considered: opposed placement, orthogonal placement, and coaxial placement. Results showed that the parametric loudspeaker can be used as a control source to mitigate sound pressure level locally without increasing noise level in other areas.

In the area of personal communication, Nakashima et al. mounted two parametric loudspeakers on a prototype mobile phone to deliver private sound field (or personal sound) to the users. Each parametric loudspeaker consists of 16 piezoelectric transducers. The SPL of the audible sound on the central axis was measured to be more than 70 dB at a distance of about 50 cm away from the phone. Because of the directivity of parametric loudspeaker, the sound pressure difference between the two ears is approximately 15 dB, which is helpful in binaural sound reproduction.

Due to the high directivity of sound waves at low frequency ranges, parametric loudspeaker has also been found to be suitable in detecting concealed objects, such as land mines or weapons. Haupt and Rolt used the parametric loudspeaker to excite the buried mines from a safe distance away. The vibration signatures of the mines were measured by the laser vibrometer to locate the position of the mines. In another work, Achanta et al. presented a novel method to detect concealed weapons by parametric loudspeakers. The metallic and non-metallic materials under clothing, as well as the abnormality in handheld devices, can be scanned and detected by the directional sound at low frequencies generated by the parametric loudspeaker system.
Sayin et al. proposed a method of utilizing parametric loudspeaker to build an omni-directional source of sound, which consists of a sphere with hundreds of ultrasonic transducers on its surface. Due to the parametric loudspeaker phenomenon, the source showed highly omni-directional for the mid-high frequency range. Moreover, the spatial correlation of the generated diffuse sound by the omni-directional parametric source diminished to zero more rapidly than using a dodecahedron source. His group also proposed a parametric loudspeaker with horns, which had obvious effects on the efficiency and directivity of the emitted sound.

Parametric loudspeaker can also be deployed to measure the acoustic performance of materials. Bernard’s group used a parametric loudspeaker and a microphone to measure the absorption coefficient of materials at normal incidence in air under the excitation of impulse signals. Kuang et al. introduced the transfer function method into the process, which allowed measurements to be taken in situ. Moreover, this method can be combined with the phase-cancellation method proposed by Kamakura to effectively remove spurious noise for accurate measurements.

7. Conclusions

This review paper outlines some of the important theoretical and experimental developments of parametric loudspeaker. An overview of numerical modelling for nonlinear acoustics, difference-frequency sound measurement, signal processing techniques, implementation issues and its applications for parametric loudspeakers is provided. There still exist some technical challenges to be overcome. For instance, although many preprocessing methods have been proposed to improve the quality of the demodulated signal, new method is still preferred, especially in the low frequency ranges. There is still a long way to go to develop a parametric loudspeaker with lower power consumption and a better quality.

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