Experimental and Spectral Finite Element Study of Plates with Shunted Piezoelectric Patches

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The need for passive damping techniques arises from the complexities, added weight, and energy requirements associated with the implementation of various active control techniques. A novel passive damping approach for the attenuation and localisation of the vibration of plates is presented in this study. The introduction of distributed piezoelectric patches with passively shunted circuits is presented. A numerical model that describes the coupling of a shunted circuit with flexible plates is developed using a spectral finite element approach. The prediction of the model is validated against analytical and experimental results. The numerical and experimental results obtained demonstrate the feasibility of using piezoelectric patches with passive shunting as an effective means for damping out the plate vibration. Furthermore, it is shown that the proposed approach is capable of producing broadband attenuation and localisation of the vibration by introducing disorder into the passive shunting circuits.

Nomenclature

A	– Area
a_i	– Undetermined coefficients of the transverse displacement shape function
b_i	– Undetermined coefficients of the electric displacement shape function
D	– Electric displacement
D_P	– Plate flexural rigidity
d	– Piezoelectric coefficient
d_i	– Nodal electric displacement
E	– Young's modulus of elasticity
$\bar{E}$	– Electric field
e	– Piezoelectric material constant relating stress to electric field
H_w, H_D	– Transverse displacement and electric displacement interpolation functions respectively
$k, k_x k_y$	– Wave number, component of wave number in x and y-directions respectively
k_b, k_D, k_{bD}	– Element bending, electric, and displacement-electric coupling stiffness matrices respectively
m_b, m_D	– Element bending and electric mass matrices respectively
N_w, N_D	– Lateral displacement and electric displacement shape functions respectively
Q	– Plane stress plane strain constitutive relation
T	– Kinetic energy
U	– Potential energy
V	– Volume
W	– External work
w	– Transverse displacement
w_b, w_D	– Nodal transverse and electric displacements respectively
$\beta = 1/\in$	
$\delta(\cdot)$	– First variation
ε	– Strain
γ_{xy}	– Shear strain
κ	– Curvature
μ	– The propagation constant
ρ	– Mass density
σ	– Stress
θ	– Wave propagation angle
v	– Poisson's ratio
ω	– Frequency
$\in$	– Dielectric constant
ϑ	– Phase angle

Subscripts

D	– Related to electric degrees of freedom
w	– Related to transverse deflection
b	– Related to bending degrees of freedom
x	– In the x-direction
$,x$	– Derivative in the x-direction
y	– In the y-direction
$,y$	– Derivative in the y-direction

Superscript

D	– At constant electric displacement
$\bar{E}$	– At constant electric field
T	– Matrix transpose

1. INTRODUCTION

1.1. Passively Shunted Piezoelectrics

In a review article about shunted piezoelectric elements, Lesieutre[1] presented a classification of shunted circuits into inductive, resistive, capacitive, and switched circuits. He emphasised that the inductive circuits, which include an inductor and a resistance in parallel with the piezo-capacitor (Fig. 1), are the most widely used circuits in damping as they are analogous to a mechanical vibration absorber.