Vibration of the London Millennium Bridge: Cause and Cure

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When the London Millennium Bridge was opened in June 2000, it swayed alarmingly. This generated huge public interest and the bridge became known as London's "wobbly bridge." Pedestrians unwittingly excited the bridge's lateral vibration modes. Although previously documented, this phenomenon was not well-known. Self-excitation occurs only when such a bridge's damping is small, so the solution was to increase damping artificially by an amount that had to be determined. This proved to be a challenging design task. This paper presents a new feedback model to describe how pedestrian motion synchronises with bridge lateral movement to become a source of self-excitation.

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1. INTRODUCTION

To mark the millennium, a new footbridge was built across the river Thames in London. It is a shallow suspension bridge linking St. Paul's Cathedral on the north side of the river with the Tate Modern art gallery on the south side. The bridge is over 300 metres long and divided into three spans, the longest being the centre span of 144 metres. To meet the designers' artistic requirements, the bridge's suspension cables sag only 2.3 metres, a fraction of the sag of a traditional suspension bridge of the same span. As a result, the cables carry a very high tension force for a bridge of this size, totalling some 2000 tonnes. When the bridge was opened, it was found to sway noticeably. With a large number of pedestrians, its sideways movement was sufficient to cause people to stop walking and hold onto the handrails. Because there was danger of personal injury, it was decided to close the bridge after a few days for remedial work.

2. HISTORY

The bridge opened on 10 June 2000. For the opening ceremony, a crowd of over 1000 people had assembled on the south half of the bridge with a band in front. When they started to walk across, with the band playing, there was immediately an unexpectedly pronounced lateral movement of the bridge deck. This movement became sufficiently large for people to stop walking to retain their balance and sometimes to hold onto the handrails for support. Video pictures showed later that the south span had been moving through an amplitude of about 50 mm at 0.8 Hz and the centre span about 75 mm at 1 Hz, approximately. Probably higher amplitudes occurred periodically and several modes were involved. It was decided immediately to limit the number of people on the bridge, but even so, the deck movement was sufficient to be uncomfortable and to raise concern for public safety. On 12 June the bridge was closed until the problem could be solved and was not reopened to the public until 22 February 2002.

There was a significant wind blowing on the opening days (force 3-4), and the bridge had been decorated with large flags, but it was rapidly concluded that wind buffeting had not contributed significantly to vibration of the bridge. Another possible explanation was that coupling between lateral and torsional deck movements was allowing vertical footfall excitation to excite lateral modes, but this was not found to be a significant factor. Early evidence in support of this conclusion was that the 1 Hz mode of the centre span, which was strongly excited, was the span's second lateral mode, which had practically no torsional movement.

It was realised very quickly that the problem was one of lateral excitation, and although allowance had been made for lateral forces, it had not been expected that pedestrians would so easily fall into step or that the lateral force per person would be as great as apparently proved to be the case.

3. RESEARCH

An immediate research programme was launched by the bridge's engineering designers, Ove Arup, supported by a number of universities and research organisations.

It was found that some similar experiences had been recorded in the literature, although these were not well known and had not yet been incorporated into the relevant bridge-building codes. A German report from 1972, quoted by Bachmann and Ammann in their IABSE book (1987), described how a new steel footbridge had experienced strong lateral vibration during an opening ceremony with 300 to 400 people. They explained how the lateral sway of a person's centre of gravity occurs at half the walking pace. Since the footbridge had a lowest lateral mode of about 1.1 Hz, the frequency of excitation was very close to the mean pacing rate of walking, about 2 Hz. Thus in this case "an almost resonating vibration occurred. Moreover it could be supposed that in this case the pedestrians synchronised their step with the bridge vibration, thereby enhancing the vibration considerably" (Bachmann, 1992, p. 636). The problem is said to have been solved by the installation of horizontal tuned vibration absorbers.

The concept of synchronisation turned out to be very important, and a later paper by Fujino et al. (1993) was discovered which described observations of pedestrian-induced lateral vibration of a cable-stayed steel box girder bridge of a size similar to the Millennium Bridge. It was found that when

Figure 1. The London Millennium Bridge after modification and shortly before final testing on a January evening in 2002. St. Paul's Cathedral is in the background.

a large number of people (some 2,000) were crossing the bridge, lateral vibration of the bridge deck at 0.9 Hz could build up to an amplitude of 10 mm, while some of the supporting cables, whose natural frequencies were close to 0.9 Hz, vibrated with an amplitude of up to 300 mm. By analysing video recordings of pedestrians' head movement, Fujino et al. concluded that lateral deck movement encourages pedestrians to walk in step and that synchronisation increases the human force and makes it resonate with the bridge deck. They summarised their findings as follows: "The growth process of the lateral vibration of the girder under the congested pedestrians can be explained as follows. First a small lateral motion is induced by the random lateral human walking forces, and walking of some pedestrians is synchronised to the girder motion. Then resonant force acts on the girder, consequently the girder motion is increased. Walking of more pedestrians are synchronised, increasing the lateral girder motion. In this sense, this vibration was a self-excited nature. Of course, because of adaptive nature of human being, the girder amplitude will not go to infinity and will reach a steady state."

Although Fujino et al. record the damping ratio of the 0.9 Hz lateral mode as $\varsigma = 0.01$, they found that only 20 percent of the pedestrians on the main span of the bridge were completely synchronised to the girder vibration when its amplitude of vibration was 10 mm (compared with 75 mm for the Millennium Bridge). Impressions from video clips of the Millennium bridge are that a good deal more than 20 percent of walkers had synchronised their step. Also in Fujino's example, the very large movement of the suspension cables (300 mm amplitude) may have made them act as dynamic vibration absorbers and so limit the extent and consequences of synchronisation.

It became clear that data specific to the Millennium Bridge was urgently required, and Arup undertook an extensive programme of testing to obtain this. In addition to commissioning tests on human gait and how this is affected by movement of the walking surface, the main tests were carried out on the bridge itself. These included artificially shaking the bridge to confirm mode shapes and damping and a comprehensive series of crowd tests. Detailed vibration measurements and video records were made with pedestrians walking at different speeds and densities on each span. These allowed reliable quantitative data on the synchronous lateral excitation phenomenon to be established and a self-excitation model to be developed which could give a reliable prediction of structural response.

4. ARUP'S PEDESTRIAN LATERAL LOADING MODEL

Arup's loading model is described in Fitzpatrick et al. (2001). Using experimental data from controlled pedestrian loading tests, with an approximately constant density of pedestrians walking at steady speed, Arup found that there was strong correlation between the amplitude of the pedestrians' (modal) excitation force and the amplitude of the bridge deck's (modal) lateral velocity. Measurement of deck velocity is straightforward, but the excitation force was calculated from a power flow analysis based on the concept that the work done by the net excitation force (footfall force less damping force) is equal to the gain of kinetic energy per cycle (see later in this paper). This analysis led to the conclusion that, when synchronisation has occurred, the amplitude of energy-transferring force per pedestrian is linearly proportional to velocity and acts as a negative damping force. This allows the limiting number of people for stability N_0 to be calculated and the effective damping to be calculated for $N \leq N_0$.

5. NEW FEEDBACK MODEL

These results can be interpreted with a completely different approach using a feedback model of synchronous lateral excitation. This interpretation is new and has not been published before. It is based on the assumption that people will naturally fall into step with each other and that they will unconsciously adjust their stepping (i.e., their phase) so that the bridge vibration increases to a maximum.

The physical mechanism of synchronous lateral excitation is well described by Arup (Dallard et al., 2001) as follows:

"Chance footfall correlation, combined with the synchronisation that occurs naturally within a crowd, may cause the bridge to start to sway horizontally. If the sway is perceptible, a further effect can start to take hold. It becomes more comfortable for the pedestrians to walk in synchronisation with the swaying of the bridge. The pedestrians find this makes their interaction with the bridge more predicable and helps them maintain their lateral balance. This instinctive behaviour ensures that the footfall forces are applied at a resonant frequency of the bridge, and with a phase such as to increase the motion of the bridge. As the amplitude of the motion increases, the lateral force imparted by individuals increases, as does the degree of correlation between individuals. The frequency "lock-in" and positive force feedback caused the excessive motions observed at the Millennium Bridge."

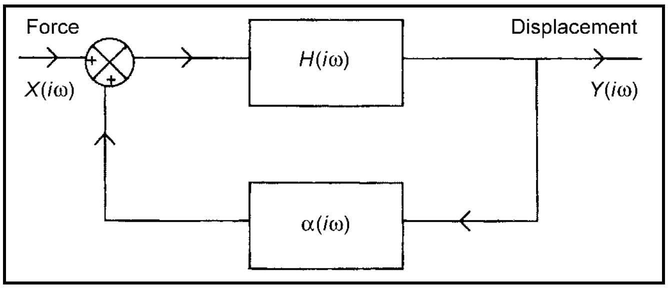

Figure 2. Feedback system to represent synchronous lateral excitation.

The frequency-domain feedback model shown in Fig. 2 represents this behaviour in a simplified way and allows some calculations to be made. We know that bridge amplitude depends on the relative phase of people's walking and the bridge's movement, and changing this phase changes the bridge's amplitude. The feedback model allows the phase to give maximum response to be calculated theoretically. It turns out that the answer agrees with the phase found by Arup et al. from their experimental studies.

Each mode is treated separately. Here $X(i\omega)$ is the Fourier transform of the modal excitation force $x(t)$ with no bridge movement, $Y(i\omega)$ is the Fourier transform of the modal displacement response $y(t)$. $H(i\omega)$ is the modal frequency response function at frequency ω and the complex quantity $a(i\omega)$ describes the positive force feedback by which the pedestrians' modal input force is modified by movement of the bridge. These are all complex quantities representing amplitude and phase at frequency ω, using the functional notation $f(i\omega)$ where $i = \sqrt{-1}$. In this notation, $f(i\omega)$ is a complex quantity (amplitude and phase) and $f(\omega)$ is a real quantity (amplitude only). The control equation is

$$Y(i\omega) = H(i\omega)\{X(i\omega) + a(i\omega)Y(i\omega)\}, \tag{1}$$

giving

$$Y(i\omega) = \frac{X(i\omega)}{\{1/H(i\omega) - a(i\omega)\}}. \tag{2}$$

The feedback function $a(i\omega)$ is a complex function which we now write as a real argument and complex exponential phase function:

$$a(i\omega) = a(\omega)\exp(-i\phi). \tag{3}$$

As noted above, the observed motion of pedestrians is that their phase adjusts itself so as to increase the motion of the bridge. Therefore it is natural to choose the phase angle ϕ so that the bridge's response is a maximum, that $|Y(i\omega)/X(i\omega)|$ is a maximum. This can be done by substituting (3) into (2) and then differentiating the denominator with respect to ϕ to search for a minimum to obtain the result that

$$|Y(i\omega)/X(i\omega)|_{\max} = \frac{|H(i\omega)|}{1 - a(\omega)|H(i\omega)|}. \tag{4}$$

For a resonant mode with modal stiffness, mass, and damping given by k, m and c,

$$H(i\omega) = \frac{1}{k + (i\omega)c + (i\omega)^2 m}, \tag{5}$$

with which Eq. (4) becomes

$$|Y(i\omega)/X(i\omega)|_{\max} = \frac{1}{|k + (i\omega)c + (i\omega)^2 m| - a(\omega)}, \tag{6}$$

When the footfall frequency ω coincides with the mode's natural frequency so that

$$\omega = \omega_n = \sqrt{k/m}, \tag{7}$$

then Eq. (6) gives

$$|Y(i\omega_n)/X(i\omega_n)|_{\max} = \frac{1}{c\omega_n - a(\omega_n)}, \tag{8}$$

or, in terms of the modal loss factor η of the bridge structure without pedestrians,

$$\eta = c/m\omega_n = c\omega_n/k, \tag{9}$$

the maximum non-dimensional response ratio is

$$|kY(i\omega_n)/X(i\omega_n)|_{\max} = \frac{1}{\eta - a(\omega_n)/k}, \tag{10}$$

In this formula, $a(\omega_n)$ is the amplitude of the modal force exerted per unit modal displacement by pedestrians walking on the bridge, when their pacing rate coincides with twice ω_n. The phasing of their movement has then naturally adjusted itself to give maximum response.

The important conclusion from this analysis is that walking pedestrians act as negative damping and the effective modal loss factor η is reduced when pedestrians walk over the bridge. This conclusion confirms that obtained by Arup from a purely experimental approach (Fitzpatrick et al., 2001, s. 4.11).

6. PHASE FOR PEAK RESPONSE

If we write

$$H(i\omega) = H(\omega)\exp(i\theta(\omega)), \tag{11}$$

then the value of ϕ for which the maximum response Eq. (4) is obtained is

$$\varphi = \theta(\omega), \tag{12}$$

which from Eq. (5) is

$$\phi = -\tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right). \tag{13}$$

So, from Eq. (3), we see that phase of the pedestrians' feedback force is leading the output displacement of the bridge deck by an angle which becomes exactly $\pi/2$ at the resonant frequency defined by Eq. (7). This of course is what we expect for a negative damping force.

7. DETERMINING THE PROPORTIONALITY CONSTANT

Referring to Fig. 2, the modal feedback force $a(i\omega)Y(i\omega)$ is generated by all the pedestrians walking on the bridge. If there are N people uniformly distributed along a span of length $z = L$ with mode shape $\Psi(z)$ normalised so that

$$\int_0^L \Psi^2(z)\,dz = L, \tag{14}$$

and if each person contributes an actual force per unit deck displacement of $\gamma(\omega)$ and per unit modal displacement of $\Psi(z)\gamma(\omega)$, the modal force from pedestrians per unit modal displacement is

$$a(\omega) = \int_0^L \Psi^2(z)\frac{N\gamma(\omega)}{L}dz = N\gamma(\omega). \tag{15}$$

Hence the net modal loss factor from Eq. (10) is

$$\eta_{net} = \eta - \frac{a(\omega_n)}{k} = \eta - \frac{N\gamma(\omega_n)}{k} = \eta - \frac{N\gamma(\omega_n)}{m\omega_n^2}, \quad (16)$$

where η_{net} is the net modal loss factor; η is the modal loss factor for the structure alone; N is the number of people on the span; L is the length of span; ω_n is the natural frequency (rad/s); k is the modal stiffness; m is the modal mass; $\gamma(\omega_n)$ is the amplitude of feedback force per person and per unit displacement of the bridge deck at frequency ω_n.

Figure 3. South and centre spans of the Millennium Bridge after its modification. The Tate Modern art gallery (in the background) is a former power station whose brick chimney is behind the bridge.

By measuring the net modal loss factor with N/L people per unit length of deck walking steadily at the synchronous speed (footfall frequency twice the natural frequency), the feedback force per person and per unit displacement, $\gamma(\omega_n)$, can be calculated from Eq. (16) if the loss factor of bare structure has been measured previously. Alternatively, if $\gamma(\omega_n)$ and η are known, η_{net} can be computed.

8. RECONCILIATION WITH ARUP'S DAMPING CALCULATION

In Fitzpatrick et al (2001, p. 20), Arup et al. give their formula corresponding to Eq. (16) as

$$c_{eff} = c + c_e = c - \frac{Nk}{8\pi fM}, \quad (17)$$

where Arup's symbols are slightly different, as follows:

Effective damping ratio	c_{eff}	=	$\eta_{net}/2$	(net loss factor)/2
Damping ratio for structure only	c	=	$\eta/2$	(loss factor for structure only)/2
Amplitude of feedback force per person and per unit velocity of the bridge deck	k	=	γ/ω_n	(amplitude of feedback force per person and per unit displacement at frequency ω_n)/ω_n
Arup's definition of modal mass	M	=	$m/2$	(modal mass used here)/2
Natural frequency (Hz)	f	=	$\omega_n/2\pi$	(angular natural frequency)/2π

Although the result Eq. (17) was arrived at by a completely different experimental approach, it is identical to Eq. (16). This can be verified by making the substitutions $\eta_{net} = 2c_{eff}$, $\eta = 2c$, $\gamma = k\omega_n$, $\omega_n = 2\pi f$, and $m = 2M$. Apart from different normalisation of the modal shape function, the main difference is that Arup et al. defined feedback force as proportional to velocity, whereas the feedback analysis in this paper begins by assuming that the feedback force is proportional to displacement (at a fixed frequency). Arup use their symbol k not for stiffness but to relate pedestrian feedback force to deck lateral velocity, whereas γ as defined above relates feedback force to deck lateral displacement.

Arup et al.'s computation of their proportionality factor k was done by measuring the acceleration time history under conditions of steady-state crowd loading with a constant number of people walking steadily over each span (in turn) at the correct speed to resonate with the relevant mode. From this time-history, they calculated modal velocity (see Fitzpatrick et al., 2001, p. 14). If F is the amplitude of the modal feedback force (which is assumed to proportional to velocity), D is the amplitude of the modal damping force (also proportional to velocity and known from previous measurements), and V is the amplitude of the modal velocity, then for conservation of energy,

$$FV/2 = DV/2 + \frac{d}{dt}(mV^2/2), \quad (18)$$

so that

$$F = D + (2m/\omega_n)\frac{dA}{dt}, \quad (19)$$

where m is modal mass, and A is the amplitude of the modal acceleration, since $A = V\omega_n$. By plotting F/N calculated from Eq. (19) (N is the number of people on the span) against modal velocity V, Arup et al. arrived at an average value for k. Some typical values are shown in Fig. 4 in which physical rather than modal results are plotted.

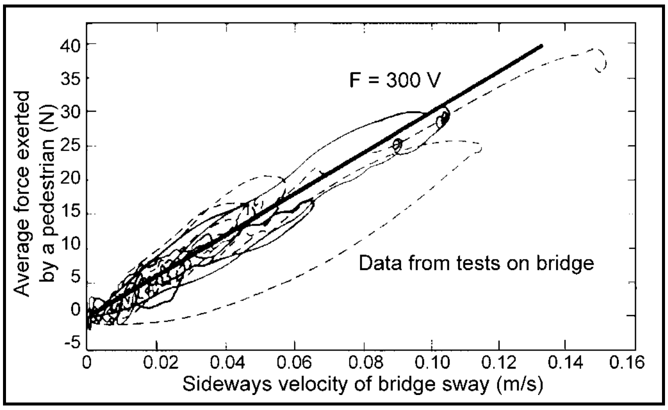

Figure 4. Correlation of pedestrian feedback force and deck velocity. The straight line shows average force exerted per pedestrian (newtons) plotted against lateral deck velocity (m/s) (Arup figure, reproduced from Deyan Sudjic (ed.), 2001, p. 93).

The approximately linear relationship in Fig. 4 appears to derive from the combined action of two factors: the force per person increases with amplitude and more people synchronise with deck movement at larger amplitudes (see the additional material in Fitzpatrick et al., 2001). Linearity is of course a starting assumption for the feedback model in part 1.

9. DEPENDENCE OF NET DAMPING ON THE NUMBER OF PEDESTRIANS

It follows from Eqs. (16) and (17) that the net loss factor will decrease in proportion to the number of walkers on the span. If N_0 is the number of pedestrians on the span when the damping decreases to zero so that $\eta_{net} = 2c_{eff} = 0$, the net loss factor for this mode when there are $N \leq N_0$ pedestrians on the span will be

$$\eta_{net} = \eta\left(1 - \frac{N}{N_0}\right). \qquad (20)$$

10. PRACTICAL DAMPING MEASURES

Based on these considerations, Arup decided to aim for 15 to 20 percent of critical damping for all lateral and lateral/torsional modes below 1.5 Hz and 5 to 10 percent of critical damping for vertical and vertical/torsional modes below 3 Hz. This is a huge increase in the original damping ratios of these modes, which were typically 1 percent or less. To understand how this was achieved, it is necessary to understand the construction of the bridge. This can be seen from the photograph, Fig. 3.

The bridge deck is carried on lateral supports spaced periodically. These reach out to clamp onto the four parallel steel cables at each side of the deck. To a first approximation, the bridge vibrates like a taut string passing over supports at the two bridge piers and anchored to fixed supports at the river banks. Therefore lateral vibration involves shearing of the deck structure with no appreciable bending. All the low frequency modes have nodes at the attachment of the cables to the bridge piers and to the river bank anchorages. Although linear viscous dampers can be connected between the bridge deck and these fixed anchorages, the relative motion here is small and dampers fixed here cannot be made to work efficiently. Maximum relative shear displacement occurs between the lateral supports near antinodes, and therefore away from the fixed anchorages.

The solution adopted was to fit A-shaped frames to alternate lateral supports, with the points of two As meeting at the intermediate supports (as shown in Fig. 5). Between the points of each pair of A frames, a linear viscous damper was mounted. It was possible to do this so that the moving parts were supported vertically on the upper side of the lateral supports. All the viscous dampers were supplied by the U.S. firm Taylor Devices, Inc. and incorporated metal bellows seals so that they are fully sealed to the environment.

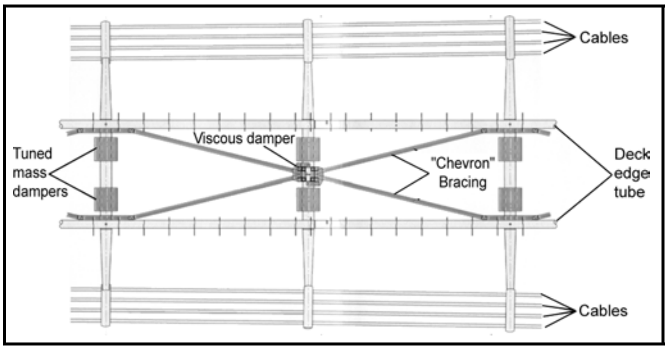

Figure 5. Plan view of underside of deck showing installation of dampers (from Fitzpatrick et al, 2001).

For the centre span, the damping introduced by frame-mounted viscous dampers was supplemented by the action of four pairs of laterally acting tuned-mass vibration absorbers supplied by the German firm Gerb Schwingungsisolierung GmbH, mounted on the upper side of the bridge deck's lateral supports, as shown in Fig. 5.

An additional 26 pairs of vertically acting tuned-mass vibration absorbers were installed in similar positions on other lateral supports to increase vertical damping. This is to guard against the (unlikely) possibility that synchronous vertical vibration might occur when the lateral problem had been removed. The tuned-mass vibration absorbers have masses between 1 and 3 tonnes and they are located as close as possible to the antinodes of the modes that they are damping.

11. EXPERIMENTAL VALIDATION

In addition to a range of laboratory tests to study human gait and the interaction of pedestrians and moving platforms, the main experimental tests were carried out on the bridge itself. These consisted of two essentially different types of tests. Tests with no people, using a mechanical shaker to provide excitation, were carried out to measure modal frequencies and damping. This was done initially for the bare bridge, and then for the bridge with specimen viscous and tuned-mass dampers installed, to verify their action. Tests with walking people consisted mostly of recirculating tests where a metered number of pedestrians walked in one direction across a single span, and then immediately turned round and walked back to their starting point.

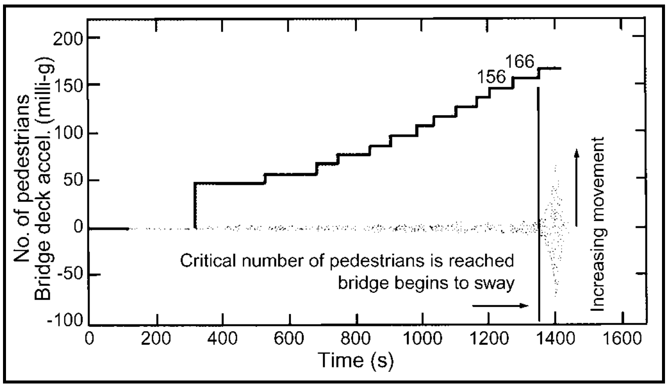

Figure 6. Onset of instability in crowd test on undamped north span, fundamental mode. As the number of people walking on the span (upper graph) increased to 166 progressively, the bridge lateral acceleration (lower graph) increased only slowly until instability was reached. The peak acceleration reached was about 80×10^{-3} g at the righthand side (Arup figure from Deyan Sudjic (ed.), 2001, p. 93).

Results from these tests were used to generate data like that in Fig. 4 and to confirm the essentially unstable feature of lateral synchronous excitation. A typical result for the north span, without any added damping, is shown in Fig. 6. A metered number of people were instructed to walk steadily at the speed needed to synchronise with the first lateral mode of the north span. Progressively the number of people walking was increased as shown by the staircase graph. The bridge deck acceleration (plotted below the staircase graph) increased slightly until 166 people were walking, when there was a sudden increase in deck lateral response which was sufficiently violent to stop the test. Since, when fully-laden, the north span can accommodate perhaps 700 people, the reason for the problems on opening day is apparent.

The performance targets for the modified bridge were expressed as rms acceleration levels measured at the quarter and half-span points with a 1 minute averaging time. The lateral target, after filtering with a passband of 0.2 to 2.4 Hz was that the rms should not exceed 25×10^{-3} g laterally; the vertical target in a passband of 0.2 to 4.8 Hz was that the rms should not exceed 50×10^{-3} g vertically. These targets were to be met in the presence of a test in which 2,000 people walked over the bridge three times at 0.6, 0.9 and 1.2 m/s approximately with the bridge comfortably full of people. A great deal of planning went into the organisation and implementation of this test, which was successfully completed on 30 January 2002. Measured acceleration levels were substantially below the target limits for all the tests, typically less than one sixth of the agreed limits.

12. CONCLUSIONS

The introduction of damping by a combination of frame-mounted viscous dampers and tuned-mass vibration absorbers has cured the London Millennium Bridge's famous wobble. It was caused by synchronous lateral excitation from pedestrians, a phenomenon that was not well-known at the time but for which there is now a good understanding and good data. Two detailed papers by Ove Arup about the bridge are listed below. The interpretation of experimental results in terms of the feedback model described in this paper is published for the first time.

ACKNOWLEDGEMENTS

The bridge's engineering designers were the Ove Arup Partnership and they designed, tested and supervised the construction of the vibration control system described in this paper. The author's role was as independent technical advisor to the London Millennium Bridge Trust (the bridge's principal funding body) for the duration of the remedial work described above. During this period, July 2000 to January 2002, he was pleased to work with the Ove Arup Partnership, and with the W. S. Atkins Group who were advising the London Borough of Southwark and the Corporation of London. A preliminary version of this paper was presented at the Ninth International Congress on Sound and Vibration, Orlando, 2002.

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