Acoustic Energy Concept for the Design of a Flow Meter

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An acoustic energy source concept is used to develop simple theoretical expressions for the optimisation of the dimensions for the design of a flow meter. Basic equations are first generated for the prediction of vortex shedding frequency due to a bluff body placed inside a pipe or a channel flow. The Strouhal number predicted using these expressions is compared with limited experimental data for a 6-inch commercial vortex flow meter operating at different Reynolds numbers. The results show good agreement between theory and experiment.

NOMENCLATURE

a — speed of sound
A — a non-dimensional parameter
C — characteristic length of a bluff body shedding vortices
d — diameter of a circular cylinder
D — diameter of pipe or channel width
f — vortex shedding frequency
N — an integer
P — perimeter of the vortex shedding body
S_R — Strouhal number ($= fd/U_\infty$)
U_V — velocity associated with vortex street
U_∞ — free stream velocity
X — spacing between two consecutive vortices of the same row
Y — spacing between two parallel rows of vortices
K — circulation of vortices

1. INTRODUCTION

Measurements in fluid mechanics are carried out using a multitude of techniques.[1-3] The measurement of flow rates using devices commonly known as 'flow meters' is one of the most common measurements made in flowing fluids, such as in pipes. Flow meters, however, use a variety of different principles to measure volume flow rate. One such principle is vortex shedding caused by a bluff body. It has long been known[4] that the sound frequency produced by a translating cylindrical rod is related to a characteristic length of the body and the velocity of the flow. There are several commercial flow measurement devices that use this principle to quantify mass or volume flow rates through pipes or channels. There is a general consensus that vortex shedding is the consequence of instability in the boundary layer on a body and is dependent on the Reynolds number of the flow. The treatment of boundary layer, however, poses formidable problems, conceptually, theoretically and experimentally.[5-8] Consequently, the design and development of vortex meters has evolved by trial and error or in an empirical manner.

In this paper, an alternative approach is explored which can produce easy to use expressions that can be exploited in the design of a stable and linear vortex flow meter. The approach is based on acoustic energy considerations. As far as the author is aware, this is the first and novel application of such an approach in the design of a flow meter.

2. BASIC CONCEPTS

The problem of vortex shedding can be viewed as an acoustic energy effect in the wake.[9-11] The basic assumption behind this consideration is that the velocity fluctuations in a turbulent wake and the fluctuating surface pressure stresses at the body surface act as acoustic sources. The sound intensity of these sources varies to the eight power of the relative fluid velocity. For alternate vortex shedding such as on a circular cylinder, three centres of disturbances can be identified: two strong areas at the separation points and a weak one at the stagnation point. Acoustic pressure pulse sources can then be considered concentrated at these centres. The frequency of the vortex pair can be assumed to be given by:[12]

$$f = \frac{3}{2P}(U_\infty - U_V). \tag{1}$$

No practical method was suggested in reference[12] to determine U_V. Consequently, this expression has rarely been used.

Using well known hydrodynamic principles, however, a theoretical expression of U_V can be obtained for two parallel rows of vortices of the same spacing, X, but of opposite circulation K. The vortices are so arranged that each vortex of the upper row is directly above the mid point of the line joining two vortices of the lower row and are separated by a distance Y. Under such conditions, the velocity of the vortex street, U_V is given by:[13]

$$U_V = \frac{K}{2X}\tanh\left(\frac{\pi Y}{X}\right). \tag{2}$$

When the bluff body producing alternate vortices is placed inside a pipe, the velocity contribution due to the wall effect becomes KY/XD, where D is the diameter of the pipe or width of the channel. The general stream in the distant wake assumes a velocity equal to ($U_\infty KY/XD$). Consequently, the frequency of vortex shedding can be written as:[14]

$$f = \frac{1}{X}\left(U_\infty + K\frac{Y}{XD} - U_V\right). \tag{3}$$

From Eqs. (2) and (3):

$$f = \frac{U_\infty}{X}\left[1 - \frac{U_V}{U_\infty}(1 - G)\right], \tag{4}$$

where

$$G = \frac{2}{\tanh(\pi Y/X)}\frac{Y}{D}\,. \tag{5}$$

From Eqs. (1) and (3) an expression for U_V/U_∞ can be found as:

$$\frac{U_V}{U_\infty} = \frac{1}{1+G/H}\,, \tag{6}$$

where

$$H = \frac{3X}{4P} - 1\,. \tag{7}$$

3. OPTIMISATION OF PARAMETERS FOR LINEAR AND STABLE VORTEX SHEDDING

Specific values of G and H can be found from the condition when vortex shedding is stable and linear. Examining Eq. (4), the condition of linearity can be achieved when $U_V/U_\infty = 0$ or $G = 1$. However, when U_V/U_∞, it suggests either that vortex shedding is absent, which is a trivial solution, or that a unique configuration has been achieved when the vortex street velocity is cancelled by the velocity contribution due to the wall. The latter interpretation can be satisfied when $H = 0$ in Eq. (6), giving:

$$\frac{3X}{4P} = 1\,. \tag{8}$$

Also when $G = 1$, from Eq. (5):

$$\tanh\left(\frac{\pi Y}{X}\right) = \frac{D}{Y}. \tag{9}$$

Thus Eqs. (8) and (9) give the conditions for linear vortex shedding.

In the context of the design of a flow meter, the terms P and D would be constants for a given flow meter. These terms are fixed by the geometry of the vortex shedding body and the diameter of the housing in which it is placed. Thus, for linear vortex shedding, it is observed from Eqs. (8) and (9) that both X and Y need to be constants. To satisfy this additional requirement, a particular solution is possible when vortex shedding is stable. The condition for stable vortex shedding can be obtained from the work of von Karman[15] who has demonstrated that a stable configuration for the non-symmetric vortices appearing behind a cylinder is produced when:

$$\frac{Y}{X} = 0.281\,. \tag{10}$$

However, experimental results[16-18] show that the ratio Y/X increases gradually as the vortices move downstream from the bluff body. These results are based on circular cylinder experiments. A closer examination of the vortex street behind a circular cylinder[20] suggests that the values of both X and Y are found to increase as the measurement position is moved further downstream. The increase in X is slightly larger than X in the near wake and tends towards a constant value in the distant wake. However, measurements at very large distances downstream have not proved reliable due to vortex diffusion.[20]

At different Reynolds numbers, however, the positions of separation points do not remain at the same locations on a circular cylinder. This may explain why Y/X values change in experiments that have all been carried out on circular cylinders. In the design of a commercial vortex meter, the bluff body employed generally is not a circular cylinder but takes the shape of a rectangular or a triangular body. In these configurations, the positions of the separation points can be considered fixed at the edges of the surface of the body facing the flow. Also since frequency measurements with commercial vortex meters are generally made on the bluff body itself and the measurement locations are fixed, then for stable vortex shedding, the value of Y/X can be assumed to be the constant value of 0.281 as obtained by von Karman. Thus from Eqs. (9) and (10):

$$\frac{Y}{D} = \frac{1}{2\sqrt{2}}\,. \tag{11}$$

The design optimisation task is reduced then to finding an appropriate relationship between the dimensions of the body with X and Y of the vortices. For this, the following approach is explored. It has already been mentioned in the introduction that frequency is related to the characteristic length of the body which is shedding the vortices. A non-dimensional parameter, A, can be introduced to relate, Y, the spacing between the parallel rows of vortices to this characteristic length, C, such that:

$$A = \frac{Y}{C}\,. \tag{12}$$

For a circular cylinder, its diameter d is taken to be its characteristic length, i.e., $d = C$. In fluid mechanics, the concept of the 'hydraulic diameter' is often used. The introduction of this concept would require the determination of the 'wetted perimeter' that would introduce another unknown parameter into the calculations. Consequently, but in an analogous manner, an approximate measure of the characteristic length, C, of any other bluff body configuration, is obtained by considering it to be equivalent to the diameter of a circle which has the same area as the cross sectional area, S, of the body. This gives:

$$C = \sqrt{\frac{4S}{\pi}}\,. \tag{13}$$

From Eqs. (8), (10) and (12), the term A can be obtained as:

$$A = \frac{4P}{0.281C}\,. \tag{14}$$

The term A in Eq. (12) can also be interpreted as the divergence of the streamlines from the body. Based on experimental results obtained on a circular cylinder,[20] the value of Y is expected to be different from C. The non-unitary value of A can be thought to arise from the existence of a vortex street. It may, therefore, be possible to express A as a function of U_V/U_∞. Noting that $|U_V/U_\infty|$ is expected to be small and less than one, an approximate expression, $A \approx 1 + U_V/U_\infty$, giving $U_V/U_\infty \approx (A-1)$ is suggested. Furthermore, when a bluff body is placed in a pipe, the flow area decreases and there is a proportional increase in the value of U_V giving:

$$\frac{U_V}{U_\infty} \approx (A-1)\frac{D}{D-C}\,. \tag{15}$$

Equation (15) suggests that for the same pipe or channel size, when for $A > 1$, as the characteristic length of the bluff body increases, there is a decrease in the vortex shedding frequency as given by Eq. (1). However, when $A < 1$, then for the same scenario, this would result in an increase in the vortex shedding frequency. These trends are supported by experimental results.[21]

4. COMPARISON OF THEORETICAL PREDICTIONS WITH EXPERIMENT

An attempt was made to apply the expressions developed in the theoretical model to predict the vortex shedding frequency associated with a commercial vortex flow meter. The data[22] used refers to the 6-inch vortex meter of Kent Industrial Measurement. The housing diameter, D, for this meter is 152.4 mm and the perimeter of the cross section of the vortex shedding body, P, is 120 mm. The manufacturing tolerance is ±0.01 mm. The manufacturer quoted the accuracy of their calibration data to within ±0.25%. Dividing the volume flow rate by the cross sectional area of the housing of each meter, the corresponding value of free stream velocity was found. Thereafter, theoretical frequencies were calculated using these velocities and the corresponding Strouhal numbers were determined for different Reynolds numbers. Figure 1 was drawn to compare the predicted and experimental Strouhal numbers at different Reynolds numbers for the meter. Agreement to within ±1% was obtained between the predicted and experimental results.

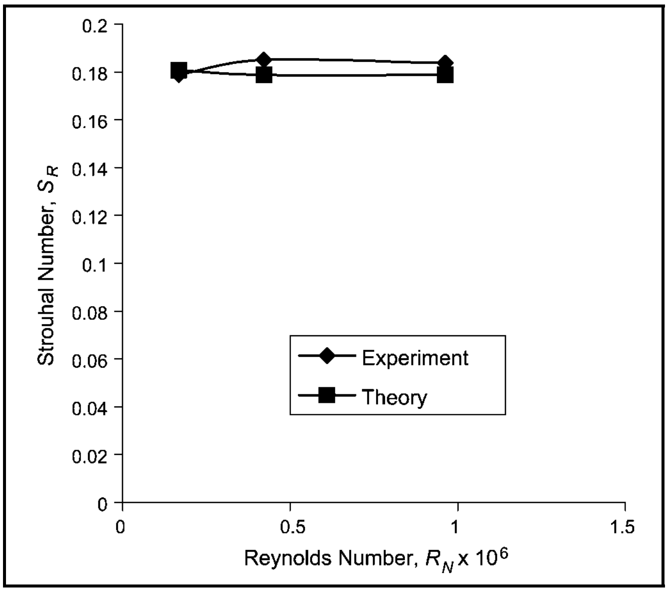

Figure 1. Comparison of theoretical prediction of Strouhal Number with experimental data[22] at different Reynolds numbers for a 6-inch vortex flowmeter.

5. CONCLUSIONS

A simple, easy to use theoretical model has been developed using an acoustic energy concept and hydrodynamic principles for the prediction of vortex shedding due to a bluff body. The procedure described here should facilitate optimisation of the vortex shedding body in relation to the pipe, in which the body can be housed, for the design of a linear and stable flow meter, without resort to extensive experimentation. The model has been validated against limited experimental data obtained with a commercial meter having a rectangular cross section.

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