Theoretical and Experimental Studies of Tubular Valve Dynamics

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Construction of a novel vibratory valve and its design optimisation is presented in this paper. The principle of the system's operation is based on the effect of the dynamic positioning of a steel ball in a vibrating tube. A theoretical analysis of the stability of this non-linear system together with an experimental study of the operating valve forms the basis of this study. Laser holographic interferometry was used for the identification and optimisation of the regimes of operation of the system.

1. INTRODUCTION

Liquid material controlled dosing and spraying has different applications – from medicine to agriculture.[1,2] Specific interest exists for elastic catheter pipe type dosing equipment. Complex dynamic processes that take place in such systems are analysed in references[3,4]. A method to study the dynamic characteristics of a tube under the influence of an internal flow is presented in reference[3]. Galerkin's method in conjunction with the method of multiple scales is employed for obtaining the stability of the tube vibration. According to the results, instability can occur under certain conditions of resonance. It is shown in reference[4] that fluid-elastic effects which are responsible for fluid-elastic instabilities may be directly measured through the analysis of the vibrating motion of a system under flow. Piezoelectric actuators are used to increase the vibratory level when buffeting forces which excite tube vibration are low, and to improve the measurement of fluid-elastic forces.

Application of piezoelectric actuators for the generation of standing waves in the outlet pipe can produce effects which can be used for the control of the dosing process. The motion of fluid-suspended particles and fibres in a standing wave field is analysed in references[5,6]. The dynamics of micron aerosol particles and their agglomeration under standing wave conditions is analysed in reference[7]. Coupling of the dynamic properties of a vibrating tube with the dynamic behaviour of a steel ball inside the tube can help producing a new type of smart doser of liquid material which can be effectively controlled by piezoelectric actuators. The unique feature of the tubular vibratory valve consists in the fact that the sealing surface of the seat is faces towards the intake duct and is located at the node of the second natural mode of transverse vibration of the elastic pipe. The vibratory valve for controlling liquid flow (Fig. 1) operates in the following fashion.

The liquid that is fed into the intake duct (4) by the force of the flow which depends on the pressure in the system brings the locking ball (6) into sealing contact with the seat (7). The valve closes and the flow of liquid through the outlet duct is interrupted. When the driving generator (6) sends control signals (the frequency of which corresponds to the second natural frequency of transverse vibration of the pipe) to the vibrator (2), the latter excites transverse vibrations in the pipe. Since the frequency of the exciting oscillations of the vibrator (2) corresponds to the second natural frequency of transverse vibration of the pipe, it initiates transverse vibration of the second natural mode at the resonance frequency. As a result, the locking element (6) overcomes the force of the flow and shifts to the point where the transverse vibration of the pipe reaches its maximum amplitude: the seat valve (7) opens and the liquid flows through the outlet duct (5). Figure 1 presents a design diagram of a vibrator valve for controlling liquid flow. It also shows the second natural mode of transverse vibration of the pipe and the location of the locking element with respect to the seat when the valve is open.

2. DYNAMICS OF A STEEL BALL INSIDE A VIBRATING TUBE

Motion of a small ball inside a tube performing transverse vibrations may be approximated by the non-dimensional differential equation of motion describing the dynamics of a mass particle on an oscillating profile with an obligatory condition of contact with the surface:[3,4]

$$\left(1+\left(\frac{\partial\zeta}{\partial x}\right)^2\right)m\ddot{x}+\frac{\partial^2\zeta}{\partial x^2}\frac{\partial\zeta}{\partial x}m(\dot{x})^2+2\frac{\partial^2\zeta}{\partial x\partial t}\frac{\partial\zeta}{\partial x}m\dot{x}$$
$$+h\left(1+\left(\frac{\partial\zeta}{\partial x}\right)^2\right)\dot{x}+\frac{\partial^2\zeta}{\partial t^2}\frac{\partial\zeta}{\partial x}m+\frac{\partial\zeta}{\partial x}mg=F, \qquad (1)$$

where $\zeta=\zeta(x,t)$ is the mode shape of the tube; $x,\dot{x},\ddot{x}$ are projections of the displacement, velocity and acceleration of the ball on the horizontal axis; t is the time; m is the mass of the ball; h is the coefficient of viscous friction between the ball and the surface of the profile; g is the acceleration of gravity; and F is the pressure force of the liquid.

Naturally, Eq. (1) is based on the assumption that the area of the cross section of the tube is small, the fluid flow is laminar, and the amplitude of the pressure of the liquid in the tube is not aligned with the amplitude of the elastic transverse vibrations. Also it is assumed that the mass of the locking ball is sufficient enough to perform the vibration-induced motion in the liquid, but not big enough to alter the shape of the tube's vibration due to its relocation.

Such a mathematical model of the system enables effective separation of the motion of the ball into relatively fast and slow motions. Co-ordinate x represents the horizontal displacement of the ball (slow motion), while the motion of the ball in the vertical direction is defined by the function $\zeta = \zeta(x, t)$.

When the tube performs steady oscillations of a standing wave type, the form of the tube displacement becomes:

$$\zeta(x,t) = b\cos(kx)\cos(\omega t), \qquad (2)$$

where b is the amplitude of oscillations, k and ω are the wavenumber and frequency of the standing wave.

2.1. Existence of Steady State Solutions

When the vibrations take place in the horizontal plane $x0y$ (Fig. 1) and no external force is applied to the mass particle, the following conditions are satisfied:

$$g = 0;\ F_x = 0;\ \dot{x} = \ddot{x} = 0. \qquad (3)$$

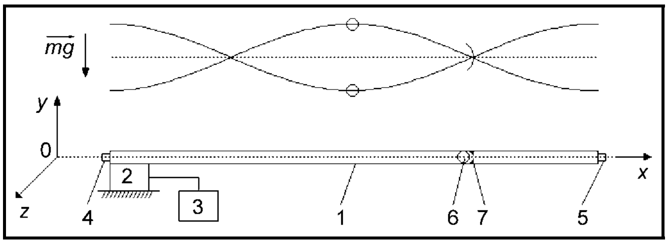

Figure 1. Structural scheme of a tubular vibratory valve in an open and closed states: 1 - the elastic pipe; 2 - the vibrator; 3 - the driving signal generator; 4 - intake; 5 - outlet; 6 - the locking element; 7 - seat.

Then, the condition of existence of the steady state solutions is found from Eq. (1):

$$\frac{\partial^2\zeta}{\partial t^2}\frac{\partial\zeta}{\partial x} = 0, \qquad (4)$$

and applying Eq. (2) to Eq. (4) :

$$x = \frac{\pi}{k}s,\ s \in Z, \qquad (5)$$

and

$$x = \frac{\pi}{2k} + \frac{\pi}{k}s,\ s \in Z. \qquad (6)$$

Similarly, if the vibrations take place in the vertical plane $x0z$ (Fig. 1), and no external force is applied to the mass particle, the condition of existence of the steady state solutions becomes:

$$\frac{\partial^2\zeta}{\partial t^2}\frac{\partial\zeta}{\partial x} + \frac{\partial\zeta}{\partial x}g = 0, \qquad (7)$$

or

$$x = \frac{\pi}{k}s,\ s \in Z. \qquad (8)$$

If the external force is non-zero, there are no steady state solutions which satisfy the condition $\dot{x} = \ddot{x} = 0$.

2.2. Stability of Steady State Solutions

Stability of solutions when the vibrations take place in the horizontal plane. The stability of the solution described by Eq. (5) can be checked by constructing the variational equation around $x = \frac{\pi}{k}s$ using the following assumption:

$$x = \frac{\pi}{k}s + dx, \qquad (9)$$

where dx is a small variation about the steady state solution.

After the assumption of approximations

$$\begin{aligned} &\sin\left(k\left(\frac{\pi}{k}s + dx\right)\right) \approx \sin(\pi s) + \cos(\pi s)\,kdx = (-1)^s kdx; \\ &\cos\left(k\left(\frac{\pi}{k}s + dx\right)\right) \approx \cos(\pi s) - \sin(\pi s)\,kdx = (-1)^s, \end{aligned} \qquad (10)$$

and the elimination of terms with $(dx)^2$ and $(d\dot{x})^2$, the following variational equation is produced from Eq. (1):

$$md\,\ddot{x} + hd\dot{x} + (bk\omega\cos(\omega t))^2 dx = 0. \qquad (11)$$

Since all coefficients of this ordinary differential equation are positive in the vicinity of t, the real parts of both roots of the characteristic equation will be negative. Thus the solution of the variational equation is stable and the position $x = \frac{\pi}{k} + dx$ is also stable. The value of the discriminant of the characteristic equation of Eq. (11) will define the type of the stable attractor. Averaging in time produces the following equation:

$$d\,\ddot{x} + \frac{h}{m}d\dot{x} + \frac{(bk\omega)^2}{2m}dx = 0. \qquad (12)$$

Therefore, the stable attractor will be of a saddle type and the roots of the characteristic equation of Eq. (12) will be real, i.e. the discriminant is positive, and an attractor of the focus type when the roots are complex (the discriminant is negative). This produces the following relations:

a) the attractor of the saddle type at $b \le \frac{h}{m\omega k}$;

b) the attractor of the focus type at $b > \frac{h}{m\omega k}$.

These conclusions agree well with the results of the numerical analysis (Figs. 2 and 3). When the parameter b is large, the absolute value of the discriminant is also large. In that case the velocities are much too larger to be linearly approximated as small values. So the attractor then turns out to be more complicated than just a point in the phase plane $(x - \dot{x})$.

The stability of the solution described by Eq. (6) can be checked in a similar manner:

$$x = \frac{\pi}{2k} + \frac{\pi}{k}s + dx, \qquad (13)$$

where dx is a small variation. Elimination of the non-linear terms and the following approximations

$$\begin{aligned} &\sin\left(k\left(\frac{\pi}{2k} + \frac{\pi}{k}s + dx\right)\right) \approx (-1)^s; \\ &\cos\left(k\left(\frac{\pi}{2k} + \frac{\pi}{k}s + dx\right)\right) \approx -(-1)^s kdx, \end{aligned} \qquad (14)$$

in Eq. (1) lead to:

$$\begin{aligned} &\left(1 + k^2b^2(\cos\omega t)^2\right)md\,\ddot{x} + \left(-m^2b^2\omega\sin 2\omega t\right. \\ &\left.+h\left(1 + k^2b^2(\cos\omega t)^2\right)\right)d\dot{x} - \omega^2b^2k(\cos\omega t)^2 mdx = 0. \end{aligned} \qquad (15)$$

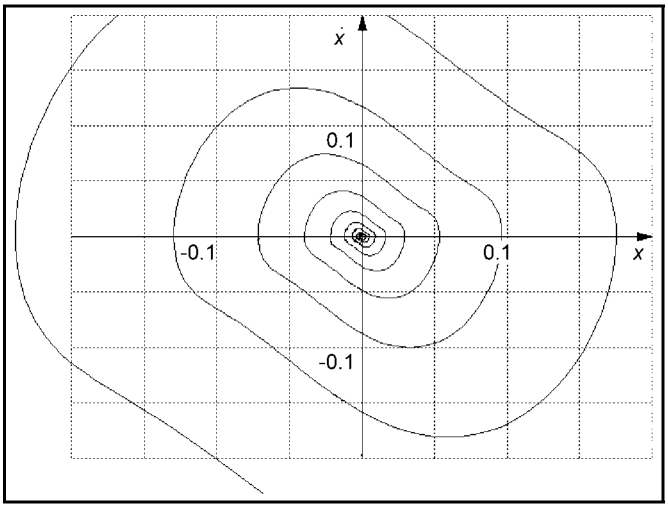

Figure 2. The attractor of the focus type at $m = 0.4$; $h = 0.1$; $k = 1$; $\omega = 1$; $b = 0.5$.

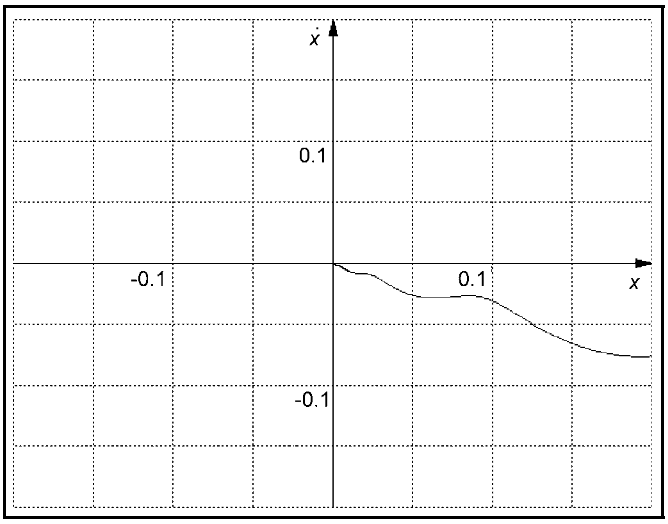

Figure 3. The attractor of the knot type at $m = 0.4$; $h = 0.1$; $k = 1$; $\omega = 1$; $b = 0.2$.

The coefficient of this ordinary differential equation at *dx* is negative in the vicinity of the local time *t*. Thus, at least one of the roots of the characteristic equation Eq. (15) will be positive, and the solution in Eq. (13) will be unstable. Thus, the positions of the ball at the nodal points of the vibrating profile are unstable. Alternatively, the positions at the peaks of the profile, i.e. at the points which vibrate with the maximum amplitude, are stable. Of course, that is true when the ball cannot jump off the surface of the profile (it is placed inside a vibrating tube).

Stability of solutions when the vibrations of the tube take place in the vertical plane. In this case the stability of the solution Eq. (8) is analysed by constructing the variational linearised equation around the steady state regime by assuming the same kind of approximations as before. The variational equation in that case is almost the same as in Eq. (11) with the exception that the coefficient at *dx* becomes:

$$(bk\omega\cos(\omega t))^2 - (-1)^s bk^2 mg\cos(\omega t)\,. \qquad (16)$$

Averaging over time in Eq. (16) shows that this coefficient is positive, thus the solution is stable. This agrees well with the data from the numerical simulations. Nevertheless, when the values of the velocity of the ball are quite large, the linearised equation cannot describe the dynamics with sufficient accuracy. Figures 4-6 show how the stable attractor turns from a point to an orbit, and consequently to a strange (chaotic) attractor.

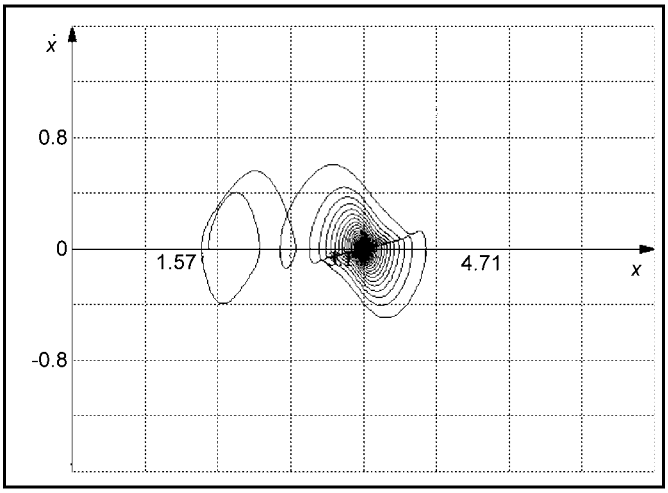

Figure 4. The attractor of the focus type at $m = 0.4$; $h = 0.1$; $k = 1$; $\omega = 1$; $b = 0.5$; $g = 0.9$.

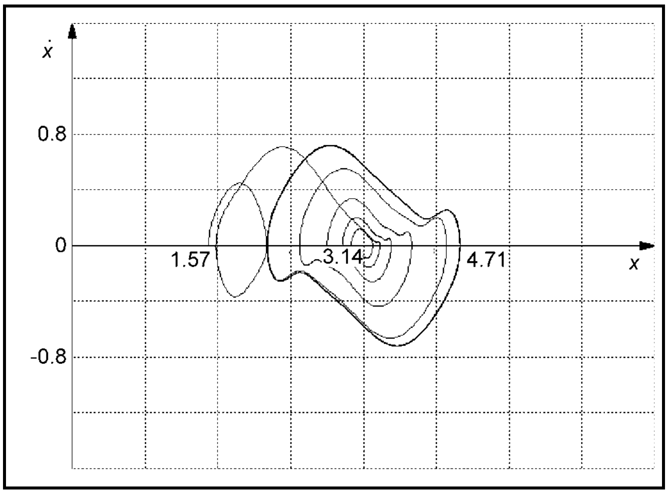

Figure 5. Stable attractor at $m = 0.4$; $h = 0.1$; $k = 1$; $\omega = 1$; $b = 0.8$; $g = 0.9$.

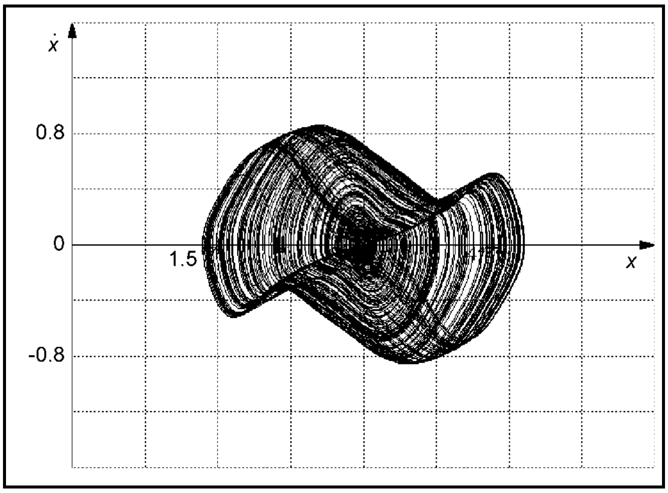

Figure 6. Stochastic attractor around the condition of open state at $m = 0.4$; $h = 0.1$; $k = 1$; $\omega = 1$; $b = 0.8$; $g = 0.9$; $F = 0.5$.

If an external force is applied, the steady state stable regimes of motion will not exist. Anyway, the stable attractor will exist, but it will form an orbit in a phase plane $(x,\dot{x})$ instead of a single point. The vibrations of the ball will take place around the peak displacements of the tube until the force F will be high enough to transport the ball along the tube.

3. EXPERIMENTAL ANALYSIS OF A TUBULAR VIBRATORY VALVE

A number of experimental studies are needed in order to ensure high dynamic accuracy of operation of the vibrator valves for controlling the flow of liquid substances. In most cases the exciting frequencies of the working tube are quite high, and the amplitudes which correspond to them are measured in micrometers. Therefore the holographic method can be effectively applied for the visual representation of wave processes taking place in the tubular vibratory valve.[1,2] The most effective method for studying the standing wave processes is the method of holographic interferometry with time averaging.[5,6] It should be noted that the most clearly visible bands in the holographic interferograms are those recorded at the positions of minimum amplitudes.[7] It is important to obtain the distribution of the vibration amplitudes not only in the middle of the dark interference bands, but also in arbitrary positions on the surface of the tube. That enables the determination of the location of the steel ball inside the tubular valve.

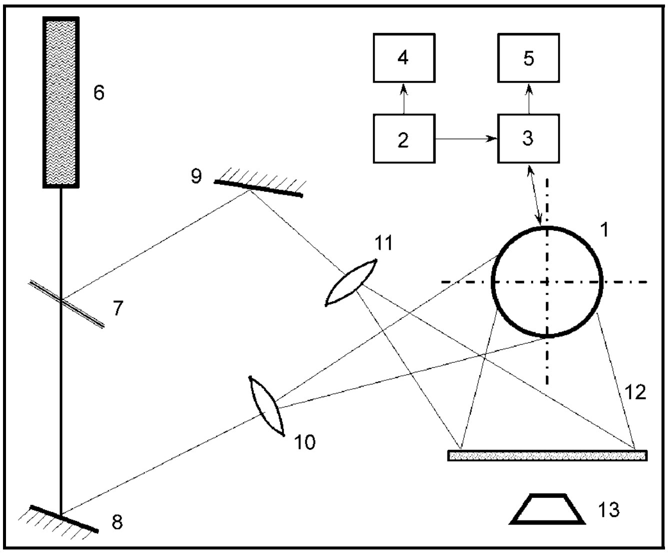

Figure 7. The structural diagram of the holography stand: 1 - the tubular working tube of a vibratory valve; 2 - the high frequency signal generator; 3 - the amplifier; 4 - the frequency meter; 5 - the voltmeter; 6 - the source of coherent radiation; 7 - the beam splitter; 8 and 9 - mirrors; 10 and 11- lens; 12 - the photographic plate; 13 - the camera.

The amplitudes of vibration of the structure were determined using the methodology presented in references[8,9]. Figure 7 presents the schematic diagram of a stand for the experimental analysis of the tubular vibratory valve. The stand contains a vibratory valve for controlling the flow of the liquid which consists of a working tube which is harmonically excited by the high-frequency signal generator (2) and the amplifier (3). The signal frequency is monitored by the frequency meter (4), the voltage amplitude of the power supply is monitored by the voltmeter (5). The optical circuit of the stand includes a holographic installation with a helium-neon laser which serves as a source of coherent radiation. The beam from the laser (6) splits into two mutually coherent beams passing through the beam splitter (7). The object beam, reflected from the mirror (8), is split by the lens (10) and illuminates the surface of the tubular working tube 1 and, after reflecting from it, impinges on the photographic plate (12). The reference beam, reflected by the mirror (9), and expanded by the lens (11), illuminates the holographic plate (12) where the interference structure is recorded.

Holographic interferograms of the transverse vibrations of a tubular vibratory valve are presented in Figs. 8(a), 8(b) and 9(a). Figures 8(a) and 8(b) make it possible to conclude that the transverse vibration of the tube is sufficiently uniform (Fig. 8(c)). Therefore, the seat of the vibratory valve can be located at a nodal point, regardless of how it is displaced lengthwise in the upper or the lower nodal point of the tube. It should be noted that the frequency of excitation must be selected with care, as the best performance of the vibratory valve takes place at resonance frequencies. If the frequency of excitation of the transverse vibrations is far away from the resonance frequencies of the tube, the operation of the tubular valve turns out to be hardly controlled at all.

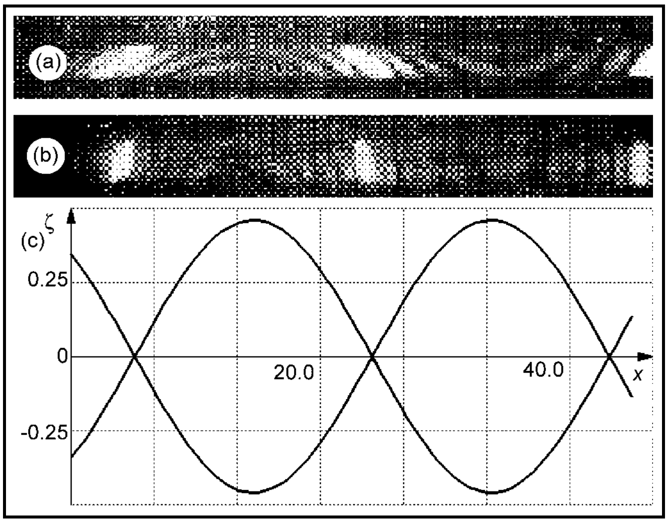

Figure 8. (a) The hologram of the transverse vibrations of the tube at $\omega = 1.3$ kHz. Illumination angle of the laser beam $\pi/4$. (b) The hologram of the tube at the illumination angle $\pi/2$. (c) The interpretation scheme of the transverse vibrations.

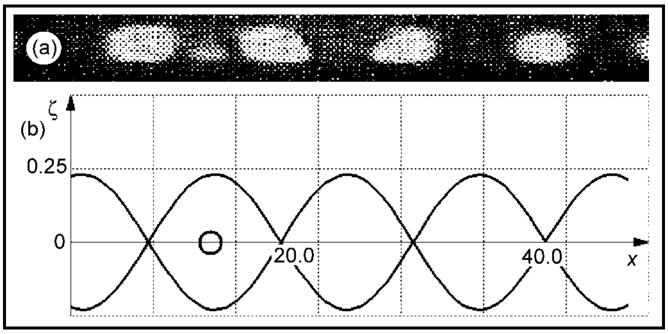

Figure 9. (a) The hologram of the tube at $\omega = 2.1$ kHz; the illumination angle of the laser beam $\pi/4$. (b) The interpretation scheme of the transverse vibrations. Circle denotes the location of the steel ball inside the tube.

The holographic interferogram presented in Fig. 9(a) corresponds to a higher frequency mode of the transverse vibrations (Fig. 9(b)). It can be seen that the modes of the transverse vibrations are shifted lengthwise as a result of the non-uniformity of the exciting oscillations of the vibrator. That influences the accuracy of locating the seat of the vibratory valve at a nodal point as well as the amplitude of oscillations (they decrease). The utilisation of those modes of motion with the corresponding characteristics makes it possible to increase the speed of action of the vibratory valves and to improve the opening capacity when the pressure forces of the liquid are increased.

The results obtained enable one to optimise the design of vibratory valves for controlling and dosing liquid flow. The following parameters of the system were analysed and optimised: a) selection of the material of the working tube; b) selection of the area of the transverse cross section of the tube; c) location of the transverse vibration nodes in the tube; d) determination of the transverse vibration amplitudes along the tube. Maximum uniformity of the transverse vibrations of the vibratory valve was achieved due to this optimisation which led to more stable operation of the whole system.

4. CONCLUSIONS

A new model of a tubular vibratory valve has been designed by using the stabilisation effect of a steel ball in the vibrating tube. The methodology of identification of the vibration peaks enabled experimental optimisation of the working regimes of the system. Such a type of analysis could be applied successfully in the design stage of different precise vibratory systems.

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