Constructive and Destructive Interference of Acoustic and Entropy Waves in a Premixed Combustor with a Choked Exit

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Thermoacoustic instabilities are a cause for concern in combustion applications as diverse as small household burners, gas turbines and rocket engines. In this work, a feedback mechanism is analysed, which couples combustion chamber acoustics with convectively transported fluctuations of entropy (entropy waves) generated within a premixed flame. Essential elements of this thermo-acoustic feedback loop are: 1) fluctuations in fuel concentration, induced by acoustic disturbances at the location of fuel injection, 2) convective transport of fuel inhomogeneities through the premixing section of the burner, 3) modulations in heat release rate and hot gas entropy resulting from the consumption of fuel/air mixture with varying fuel concentration by the flame, and 4) the generation of sound through entropy non-uniformities at the turbine inlet. From a qualitative analysis based on relative phases, it is concluded that, depending on the various convective and acoustic time lags involved, entropy waves may couple constructively as well as destructively with combustor acoustics. However, such qualitative analysis does not indicate whether the coupling between entropy and acoustic waves is strong enough to significantly influence thermo-acoustic stability. Therefore, a linear model has been constructed to estimate the effect of entropy waves on the thermo-acoustic response and stability of a combustor with a choked exit nozzle, as it might be found in a gas turbine. Note that phenomena like dispersion of convective waves, distributed heat release, vortical velocities, etc., have not been taken into account, as they would burden the presentation with unnecessary complexity. Results obtained indicate that the interaction between combustor acoustics and entropy waves can be significant, especially for the lowest non-axisymmetric modes, and even at frequencies higher than those usually associated with convective waves. As expected, it was observed that the coupling between pressure and entropy waves at the exit nozzle can enhance as well as reduce the thermo-acoustic stability of a combustor, or the responsiveness to an external or internal fluid-mechanic excitation mechanism. It is concluded that a comprehensive thermo-acoustic analysis of a premixed combustor with a choked exit must in general include the generation and propagation of entropy waves and the coupling with combustor acoustics.

1. INTRODUCTION

It has been realised quite some time ago that the flow of gas containing regions of non-uniform temperature ("entropy spots") through a nozzle can be a significant source of combustion noise.[1] A comprehensive review of earlier work on this subject has been compiled by Marble and Candel.[2]

It has also been understood that convection of entropy spots into a nozzle or more generally a choked cross section can result in self-excited combustion instability.[3-9] Much of the earlier work concentrated on ramjet combustors. See the review by Culick.[5] More recently, attention has been turned towards (premixed) gas turbine combustors; see the work by Keller et al.[4,7] and Dowling and co-workers.[6,8,9] Because entropy spots are transported by the mean flow, the frequencies associated with these instabilities have been argued to be generally "low", i.e. lower than the acoustic eigenfrequencies of the combustor.

In the present work it is emphasised that in compact premixed combustors, as they are commonly encountered in low-emission combustion systems, acoustic oscillations and entropy fluctuations couple with each other even at "high" frequencies, i.e. frequencies corresponding to acoustic eigenmodes. This coupling is achieved through a multiply-connected feedback mechanism involving fluctuations of heat-release-rate — pressure — velocity — fuel concentration and entropy. Fluctuations of entropy are generated as fuel inhomogeneities that are consumed by the flame. Note that this scenario is quite distinct from, e.g., the analysis of entropy fluctuations in a ramjet combustor by Humphrey and Culick,[10] where entropy fluctuations *upstream* of the flame drive the instability mechanism. The amplitude of these entropy waves is either imposed as an upstream boundary condition, or produced by pressure waves incident on the inlet shock.

Analysing the various convective and acoustic time lags involved in the aforementioned feedback mechanism, we arrive at a conclusion, which to our knowledge has not been emphasised previously: entropy waves may couple constructively *as well as destructively* with combustor acoustics. In other words, the interaction of pressure and entropy waves can enhance as well as reduce the thermo-acoustic stability of a combustor or the responsiveness to an external or internal fluid-mechanic excitation mechanism.

In order to illustrate our ideas and to show that the coupling between acoustic and entropy waves is of significant strength, we have examined a simple model for the propagation of acoustic and convective waves in a combustor, optionally with an internal thermo-acoustic excitation due to coherent flow structures in the region of heat release. The results obtained with the model suggest that indeed the response of the combustor to the excitation can be significantly influenced by entropy waves.

2. PHYSICAL MECHANISM AND QUALITATIVE ANALYSIS

Whenever fluctuations in heat release rate occur in a combustion system — see Fig. 1 for a simple sketch of a premixed combustor — they may couple with the system's acoustic modes. In general, oscillations of velocity and pressure will then occur also in the vicinity of the fuel injector (2 in Fig. 2). Assuming that the acoustic impedance of the fuel injector differs from that of the fuel/air mixing section (2-3), inhomogeneities in the fuel mass fraction Y_F' will result, which are convected towards the flame. As momentarily richer and leaner "pockets" of fresh mixture are consumed by the flame, the momentary heat release rate will increase and decrease, respectively. This will also cause disturbances of the momentary rate of volume production, which couples via the impedance at the combustor entrance (4) with the system acoustics. In addition to this direct, local interaction, there is a second mechanism that couples fluctuations of the fuel mass fraction Y_F' with the system acoustics. The adiabatic flame temperature depends on Y_F, and fluctuating mixture composition will result in fluctuations of hot gas temperature downstream of the flame (5), i.e. *entropy waves*. These fluctuations in hot gas entropy are convected through the combustion chamber (5-6) and couple at the choked combustor exit (6) with the system acoustics.

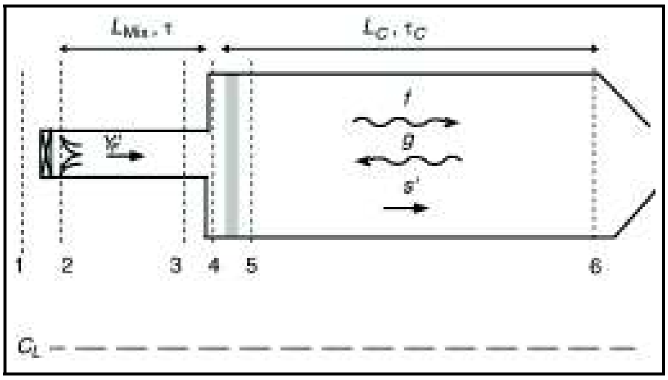

Figure 1. Sketch of a premixed combustor. 1 upstream plenum, 2 location of fuel injection, 2-3 fuel/air mixing section, 3-4 backward facing step, 4-5 heat release zone, 5-6 combustion chamber, 6 exit nozzle. The acoustic disturbances travelling in the down- and upstream direction are indicated as f and g (Riemann Invariants), the entropy wave as s.

The relative phase between the acoustic signal at the combustor exit and the pressure pulse generated by the entropy wave determines whether the susceptibility of the combustor to thermo-acoustic oscillations is enhanced or reduced by this interaction of entropy waves with combustor acoustics. This phase can be estimated in a straightforward manner by considering the various time lags involved in the propagation of sound and entropy waves.

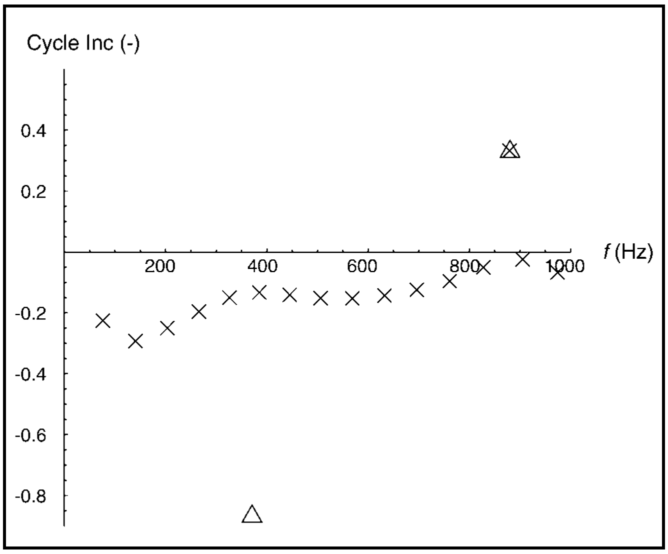

Figure 2. Cycle increment ς vs. frequency f of plane wave eigenmodes ($k_\perp = 0$) in the range 0 to 1000 Hz. Triangles — entropy waves neglected. "×" — entropy wave effects included.

Inhomogeneities in fuel mass fraction Y_F' caused by pressure fluctuations p_2' and velocity fluctuations u_2' at the fuel injector will result in fluctuations of the momentary heat release rate Q' after a time lag τ. This time lag is of convective nature, i.e. it is the time required for convective transport from the fuel injector to the region of heat release. Within the framework of a quasi-1D, linear analysis, the coupling between fluctuations in velocity and pressure at the fuel injector and heat release rate may be described by the following expression:

$$\frac{Q'_{Y_F}(t)}{Q} = -\frac{1}{2}\frac{p_2'(t-\tau)}{\Delta p} - \frac{u_2'(t-\tau)}{u(x_2)}. \tag{1}$$

Here the unprimed quantities denote mean values, and $\Delta p = p_{Nozzle} - p_2$ is the mean pressure drop across the fuel injector. The subscript Y_F is to indicate that only fluctuations of the heat release rate due to modulations of mixture composition are taken into consideration. There may be other mechanisms responsible for additional contributions to the total unsteadiness in the heat release Q' (see below).

The first term on the right hand side of Eq. (1) is due to variations in fuel mass flow rate $\dot{m}_F'$ caused by pressure fluctuations. A minus sign appears in front of the pressure term in Eq. (1), because a momentary increase of pressure in the premixing duct reduces the fuel mass flow rate,

$$\frac{\dot{m}_F'}{\dot{m}_F} = -\frac{1}{2}\frac{p_2'}{\Delta p}. \tag{2}$$

In a premixed low-emission combustion system, the pressure drop Δp is often made quite large in order to provide sufficient momentum for rapid mixing of fuel and combustion air. In this case, the fuel injection system is "stiff", i.e. the fuel injection rate does not respond significantly to acoustic perturbations, and the first term on the right hand side of Eq. (1) may be neglected.

However, even if the fuel injection system is stiff, acoustic perturbations of the *air* mass flow at the fuel injector u_2' will cause fluctuations of the fuel mass fraction Y_F' and subsequently variations in the heat release rate Q'. This phenomenon is accounted for by the second term in Eq. (1). The minus sign is appropriate here, because a momentary increase in air flow at the location of fuel injection will lower the local fuel concentration and lead to a corresponding reduction in the heat release rate. As already mentioned, this reduction does not occur instantaneously, but after a time lag τ, estimated to be equal to the residence time in the fuel-air mixing section,

$$\tau \approx \frac{L_{\mathrm{Mix}}}{u_2}. \quad (3)$$

For harmonic perturbations $\sim \exp(i\omega t)$ with frequency ω the corresponding phase lag ϕ_Q relative to u_2' equals

$$\phi_Q = \pi - \omega\tau, \quad (4)$$

where the factor π is due to the minus sign in Eq. (1). The generation of entropy waves, i.e. temperature inhomogeneities, in the region of heat release is in phase with Q'_{Y_F}. As temperature inhomogeneities are convected downstream, they undergo a phase change equal to $-\omega\tau_C$, where $\tau_C = L_C/u_h$ is the combustor residence time.

At the combustor exit, the flow accelerates into the turbine, and the entropy wave produces a sound signal which is reflected back into the combustor. At least for small Mach numbers within the combustion chamber, the turbine inlet appears, to acoustic waves, similar to a closed end, i.e. incoming acoustic waves are reflected without phase change. On the other hand, one can show (see Eq. (42) below) that the acoustic signal produced by an incoming entropy perturbation is usually approximately opposite in phase to the entropy wave. To summarise, the overall phase difference between entropy perturbations generated at the flame and the acoustic signal generated by these perturbations at the combustor exit is approximately equal to

$$\phi_s \approx -\omega\tau_C + \pi. \quad (5)$$

It remains to account for the phase lag ϕ_f between pressure disturbances at the combustor exit and the velocity fluctuations u_2' at the fuel injector. If a standing wave pattern with upstream and downstream travelling waves of equal amplitudes is established in combustor and burner, this phase difference will be close to $\pm\pi/2$. Its exact value will depend on the location of the fuel injector and combustor exit with respect to the wave pattern. However, if losses occur within the system, or other than perfectly reflecting boundary conditions are present, ϕ_f cannot be determined accurately without detailed analysis of the wave fields. Further complications arise with the phase shifts generated by the change in cross sectional area between locations 3 and 4, which acts as an element of negative or positive *virtual length*, depending on the value of the acoustic impedance at the area change.

Finally, the following estimate for the relative phase ϕ_{total} between the reflection of f_6 and the pressure signal g_s generated by the interaction of the entropy wave and the nozzle is obtained:

$$\phi_{total} = \phi_Q + \phi_s + \phi_f = -\omega(\tau + T_C) + \phi_f. \quad (6)$$

In the limit of linear acoustics, the acoustic and entropy wave processes can simply be superposed upon each other, and one may conclude that entropy waves will tend to enhance the susceptibility of the combustor to thermo-acoustic oscillations whenever the relative phase ϕ_{total} is near π (or an even multiple of π). More precisely, constructive interference between entropy and acoustic waves is expected for $-\pi/2 < \phi_{total} < \pi/2$. If this condition is not fulfilled, *destructive* interference should be expected; in this case the entropy disturbances will enhance the thermo-acoustic stability of the combustion system.

However, it is not clear from the above considerations based solely on relative phases, whether the coupling mechanism described will be of sufficient strength to significantly influence the thermo-acoustic characteristics of a combustion system. A simple quantitative model of a combustion system as shown in Fig. 1, which has been developed to answer this question, is described in the next Sections. First we derive the transfer functions for acoustic fluctuations of velocity u' and pressure p' across a thin flame sheet from the conservation equations for mass, momentum and energy. It is found that these coupling relations include terms which depend on fluctuations of the heat release rate Q'. Closure for the transfer functions is achieved with the help of Eq. (1), which describes how fluctuations of the heat release rate depend on acoustic perturbations and the mean flow field at the fuel injector and in the premixing section, respectively. In addition, a model for acoustic driving due to coherent vortical structures is proposed. The source term for entropy waves resulting from fuel modulations is then introduced, followed by a brief description of the exit boundary conditions. Finally, both the thermo-acoustic stability of the homogeneous system as well as the acoustic response of the system to an inhomogeneity is computed with and without consideration of entropy waves.

3. LINEARISED RANKINE-HUGONIOT RELATIONS FOR ACOUSTIC FLUCTUATIONS

In this Section, the relations coupling fluctuations of velocity and pressure across a compact premixed flame are derived. Following Chu[11] and Keller et al.,[4,7] the premixed flame is considered as a *flame sheet*, i.e. as a discontinuity of negligible thickness where heat is added to the otherwise isentropic flow of an ideal gas.

In the steady case, conservation of mass, momentum and energy across a flame sheet with a given rate of heat addition per unit mass q may be written as

$$[\rho u] = 0; \quad (7)$$

$$[p + \rho u^2] = 0; \quad (8)$$

$$\left[h + \frac{1}{2}u^2\right] = q, \quad (9)$$

where $[x] := x_h - x_c$ is the difference of a quantity x across the discontinuity. The subscripts c and h refer to the cold and hot states, respectively. To close this system of equations, further relations are needed.

The equation of state of a perfect gas is

$$\frac{p}{\rho} = RT, \quad (10)$$

where R denotes the gas constant, (J/kg·K). Combining this with the definition of the speed of sound c in isentropic flow,

$$c^2 = \frac{\gamma p}{\rho}, \tag{11}$$

one obtains

$$\left(\frac{c_h}{c_c}\right)^2 = \frac{T_h}{T_c}, \tag{12}$$

if it is assumed that $[R] = [\gamma] = 0$.

For an ideal gas, the enthalpy $h = c_P T = e + p/\rho$ with $e = c_V T$ may be expressed in terms of pressure and density as

$$h = \frac{\gamma}{\gamma - 1}\frac{p}{\rho}, \tag{13}$$

where γ is the ratio of specific heats $\gamma = c_p/c_v$.

Employing Eqs. (11) and (13), the energy equation may be rewritten as

$$\left[\frac{c^2}{\gamma - 1} + \frac{1}{2}u^2\right] = q. \tag{14}$$

Expanding this expression in terms of the Mach number $M = u/c$ one obtains:

$$\left(\frac{c_h}{c_c}\right)^2 = 1 + \frac{\gamma - 1}{\gamma}\frac{Q}{p_c u_c} + O(M^2), \tag{15}$$

where $Q = \rho u q$ is the rate of heat addition per unit area. Similarly, if the ratio of pressure and density is expressed in terms of the speed of sound and the ratio of specific heats, then the momentum equation yields

$$\frac{u_h}{u_c} = 1 + \frac{\gamma - 1}{\gamma}\frac{Q}{p_c u_c} + O(M^2). \tag{16}$$

Alternatively, one can eliminate velocity and density from the momentum equation to derive a relation for the pressures:

$$\frac{p_h}{p_c} = 1 - (\gamma - 1)\frac{Q}{p_c u_c}M_c^2 + O(M^4). \tag{17}$$

Note that combining Eqs. (12) and (15) yields the useful relation

$$\frac{T_h}{T_c} - 1 = \frac{\gamma - 1}{\gamma}\frac{Q}{p_c u_c}, \tag{18}$$

which shall be used in the following.

Now consider the presence of (small) acoustic fluctuations, i.e. $p \to p + p'$, where $p' \ll p$ and similarly for all other quantities of interest, i.e. u, c, ρ, T and Q. The relations between fluctuating quantities up- and downstream of the flame sheet are obtained by expanding the relations Eqs. (15)-(17) and keeping only terms of first order in acoustic quantities. For the pressure, one may proceed as follows. First rewrite Eq. (17) as

$$p_h = p_c - (\gamma - 1)\frac{Q u_c}{c_c^2}. \tag{19}$$

Then introduce fluctuations p', u', c' and Q'. Using relations $c^2 = \gamma p/\rho$, Eq. (18) and

$$\frac{c'}{c} = -\frac{\gamma - 1}{2}M^2\frac{u'}{u}, \tag{20}$$

which follows from the compressible steady Bernoulli equation, the following coupling condition for velocity fluctuations is derived:

$$u_h' = u_c' + \left(\frac{T_h}{T_c} - 1\right)u_c\left(\frac{Q'}{Q} - \frac{p_c'}{p_c}\right), \tag{21}$$

which is accurate to first order in Mach number. The corresponding expression for pressure fluctuations p' is obtained in a similar manner:

$$p_h' = p_c' - \left(\frac{T_h}{T_c} - 1\right)\rho_c u_c^2\left(\frac{u_c'}{u_c} + \frac{Q'}{Q}\right). \tag{22}$$

It is easy to show that the pressure drop (the second term on the right hand side of the above equation) itself is of first order in Mach number, since deviations from Eq. (22) are second order in Mach number.

With the important exception of the terms involving fluctuations of the heat release Q', we have reproduced here the equations given by Keller.[7] Note that it is assumed that entropy inhomogeneities are absent upstream of the zone of heat release.

To achieve closure for the flame transfer matrix, assumptions concerning the response of the heat release rate to acoustic fluctuations must be made, so that the variations of heat release rate Q' can be expressed in terms of the fluctuations of velocity u' or pressure p'. The closure model used here is based on time-lagged equivalence ratio modulations as described by Eq. (1). Fluctuations of velocity at the fuel injector at an earlier time $t - \tau$ modulate the momentary rate of the heat release rate $Q_{Y_F}'(t)$. Without essential loss of generality, it is assumed that the fuel injector is stiff and neglect the first term on the right hand side of Eq. (1). This closure model leads to a homogeneous system of equations (see below), comprising several feedback loops between pressure, velocity, heat release rate, fuel concentration and entropy.

In addition to heat release rate modulations caused by equivalence ratio variations, we consider fluctuations in the momentary consumption rate due to the existence of coherent vortices in the flow, which modulate the rate at which fresh mixture and hot gas are mixed and subsequently burned. For this mechanism, we use the simple model proposed by Keller:[7]

$$\frac{Q_\Omega'}{Q} = \epsilon e^{i\omega t}, \tag{23}$$

which provides acoustic driving by the flame. Here the parameter ϵ stands for the fraction of the total fuel, perhaps a few percent, which is entrained by coherent structures and consequently releases its heat of reaction in an unsteady manner. This is admittedly a very crude model, and since the original publication of the ideas presented here,[12,13] a more elaborate formulation for an acoustic source term due to coherent structures has been developed and experimentally verified by Paschereit et al.[14] This formulation could also be used here without essential difficulty. However, the purpose of the present analysis is to gain qualitative insight into the interactions of entropy waves with combustor acoustics. In this respect, the present work would not benefit from a more complicated formulation for the driving term and therefore the simple model Eq. (23) is used. Note that if the term Eq. (23) is introduced, the resulting system of equations is *inhomogeneous*. Rather than determining the stability of the system with respect to self-excited instabilities, its *response* to the inhomogeneity is then the quantity of interest.

4. SOURCE TERM AND BOUNDARY CONDITION FOR ENTROPY WAVES

Heat release rate fluctuations Q'_{Y_F} due to fluctuations in the fuel mass fraction Y'_F result in fluctuations of the hot gas temperature T_h and therefore lead to the generation of entropy non-uniformities. These entropy non-uniformities trigger acoustic disturbances when convected into the exit section of the combustion chamber.

The second law of thermodynamics $s' = c_p T'/T_h$ combined with Eq. (1) gives the connection between acoustics and the generation of entropy non-uniformities at the location of the flame:

$$s' = \frac{c_p T'}{T_h} = c_p\left(1 - \frac{T_c}{T_h}\right)\frac{Q'_{Y_F}}{Q} = -c_p\left(1 - \frac{T_c}{T_h}\right)\frac{u'_c(x_2, t-\tau)}{u_c(x_2)}. \tag{24}$$

Note that here Mach number corrections are neglected, which is usually appropriate.

Coupling between entropy disturbances and the combustion chamber acoustics can occur whenever a pressure gradient is present, e.g. in regions of varying cross-section, or at a critical cross section. Marble and Candel[2] have shown that the following relation holds on both sides of a compact critical nozzle:

$$\frac{u'}{c} - \frac{\gamma M}{2}\frac{p'}{\gamma p} + \frac{M}{2}\frac{\rho'}{\rho} = 0. \tag{25}$$

Linearising $p = \rho^\gamma \exp\{(s - s_0)/c_v\}$, one obtains

$$\frac{\rho'}{\rho} = \frac{p'}{\gamma p} - \frac{s'}{c_p}. \tag{26}$$

Combining the last two equations yields a coupling between entropy and acoustics as well as the required boundary condition for the acoustics at the exit 6 of the combustor:

$$u'_6 = M_6 \frac{\gamma - 1}{2}\frac{p'_6}{\rho_h c_h} + \frac{u_h}{2}\frac{s'_6}{c_p}, \tag{27}$$

where M_6 denotes here the Mach number just upstream of the nozzle.

5. HIGHER ORDER MODES

The analysis as it has been presented so far is not restricted to plane waves. Higher order modes in rectangular, cylindrical or annular cavities may be discussed within the present framework by introducing a perpendicular wave number $k_\perp$. For example, consider the structure of the eigenmodes in a three-dimensional cavity of rectangular cross-section of width a and height b. The pressure distribution of the mode (m, n) in such a cavity is given by Munjal[15]

$$p'(x, y, z) = (Ae^{-ik_x x} + Be^{ik_x x}) \cos\left(\frac{m\pi}{a} y\right) \cos\left(\frac{m\pi}{b} z\right) \tag{28}$$

with

$$k_x^2 = \left(\frac{\omega}{c}\right)^2 - k_\perp^2; \tag{29}$$

$$k_\perp^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2. \tag{30}$$

Similar expressions may be derived for the cases of cylindrical or annular combustor geometries.[15] Also, the effects of mean flow — which are of first order in the mean flow Mach number — on the propagation of acoustic waves may be included without essential difficulty. This has not been done in the present analysis in order to allow for a more straightforward presentation of the main results.

Using axial and perpendicular wave numbers k_x and $k_\perp$, one can describe the pressure and velocity distribution of non-plane modes in cylindrical or annular cavities. However, what are the proper boundary conditions in this situation? In annular combustors, as they are typically found in modern gas turbines, it is in general true that the height of the combustion chamber, the diameter of the burners, and the width of the flow passages between the turbine inlet guide vanes are much smaller than the circumference of the combustor. It follows that radial modes need not be considered, and that for the lowest azimuthal modes the burners and the turbine inlet passages are *azimuthally compact*, i.e. their dimensions in the tangential direction are much smaller than the wavelength of the azimuthal modulation of the pressure and velocity field. One may conclude that the relation Eq. (27) is also appropriate as the downstream boundary condition for azimuthal waves in an annular combustor, and similarly for the burner on the upstream side of the combustion chamber. As one of the reviewers has pointed out, use of the relation Eq. (27) is not permitted in the case of a radial or circumferential mode in a cylindrical duct terminated by a single nozzle.

6. SIMPLE QUANTITATIVE MODEL OF A PREMIXED COMBUSTOR

Using the relations established in the above sections, a simple yet quantitative model of a premixed combustor as shown in Fig. 1 is now constructed and subsequently its thermo-acoustic properties are analysed. The goal of the analysis is to establish whether interference of acoustic and entropy waves can be strong enough to have an effect on the thermoacoustic stability or response of a combustion system, and to check to what extent the simple stability arguments based on relative phases are confirmed by the model's behaviour.

From a plenum 1 air enters a fuel injection section 2, passes through a mixing section of length L_{Mix} until it encounters at 3 an expansion in cross sectional area with the area ratio $\alpha \equiv A_3/A_4$. The flame front is located between 4 and 5. Within a short distance downstream of the combustor exit 6, a compact choked nozzle is located.

For matters of convenience, the model is formulated in terms of the *Riemann invariants f* and *g* instead of the fluctuations of pressure p' and velocity u'. The Riemann invariants f, g — sometimes also referred to as p'^{+} and p'^{-} — are fundamental solutions of the wave equation and may be interpreted as acoustic waves travelling in the positive (downstream) or negative (upstream) direction, respectively. Note that the following relations hold between fluctuations of velocity u' and pressure p' and the Riemann invariants

$$\frac{p'}{\rho c} = f + g\,; \tag{31}$$

and

$$u' = \chi(f - g)\,, \tag{32}$$

where $\chi = \sqrt{1 - (k_{\perp}/k)^2}$.

Using the Riemann invariants, the combustion system sketched in Fig. 1 can be modelled as the following set of equations:

- At location 1, assume (for simplicity) that in the burner plenum $p'_1 = 0$.
- At location 2, the difference in mean pressure between the burner plenum and the location of fuel injection (2) can be described by

$$p_2 - p_1 = -\frac{\zeta}{2}\rho u_2^2\,,$$

where ζ is an appropriate pressure loss coefficient. Linearising this relation yields in combination with the plenum condition at 1

$$f_2 + g_2 = -\zeta M_2 \chi_2 (f_2 - g_2)\,. \tag{33}$$

Note that with this formulation, the variables f_1, g_1 in the burner plenum do not appear at all in the system of equations representing the combustor.
- From locations 2-3:

$$\begin{pmatrix} f_3 \\ g_3 \end{pmatrix} = \begin{pmatrix} e^{-ik_x L_{\text{Mix}}} & 0 \\ 0 & e^{+ik_x L_{\text{Mix}}} \end{pmatrix} \begin{pmatrix} f_2 \\ g_2 \end{pmatrix}, \tag{34}$$

where L_{Mix} is the length of the fuel-air mixing tube. Note that in this expression, $k_x = \omega/c_c$ for plane as well as non-plane waves, because by assumption the mixing tubes are azimuthally compact.
- At location 4, conservation of mass and momentum across the change in cross-sectional area yields:

$$\begin{pmatrix} f_4 \\ g_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1+\alpha & 1-\alpha \\ 1-\alpha & 1+\alpha \end{pmatrix} \begin{pmatrix} f_3 \\ g_3 \end{pmatrix}. \tag{35}$$

- At location 5, the Rankine-Hugoniot relation Eq. (22) for pressure yields:

$$\xi(f_5 + g_5) = f_4 + g_4 - \left(\frac{T_h}{T_c} - 1\right) M_4 \times$$
$$[u_4 \in - \alpha e^{-i\omega\tau} \chi_2 (f_2 - g_2) + \chi_4 (f_4 - g_4)]\,, \tag{36}$$

with $\xi = (\rho_5 c_5)/(\rho_4 c_4)$ being the ratio of specific impedances. Note that the relation $u_4/u_2 = \alpha$, which couples the *mean* velocities on the cold side of the flame and at fuel injection, has been employed. Correspondingly for velocity

$$\chi_5 (f_5 - g_5) = \chi_4 (f_4 - g_4) + \left(\frac{T_h}{T_c} - 1\right) \times$$
$$[u_4 \in - \alpha e^{-i\omega\tau} \chi_2 (f_2 - g_2) - \gamma M_4 (f_4 + g_4)]\,. \tag{37}$$

For the entropy, Eq. (24) yields:

$$\frac{u_5}{c_p} s'_5 = -\alpha \left(\frac{T_h}{T_c} - 1\right) e^{-i\omega\tau} \chi_2 (f_2 - g_2)\,. \tag{38}$$

- From locations 5-6:

$$\begin{pmatrix} f_6 \\ g_6 \end{pmatrix} = \begin{pmatrix} e^{-ik_x L_C} & 0 \\ 0 & e^{+ik_x L_C} \end{pmatrix} \begin{pmatrix} f_5 \\ g_5 \end{pmatrix}, \tag{39}$$

for the acoustic waves, determining k_x from Eq. (29) with c_h as the appropriate value for the speed of sound. For the entropy, there is simply a change in phase due to convective transport,

$$s'_6 = e^{-i\omega\tau_C} s'_5\,, \tag{40}$$

where it has been assumed that the convective velocity within the combustion chamber does not vary significantly away from the nozzle and is equal to the velocity u_5 right after the flame front.
- Finally, the nozzle exit condition at 6 is:

$$\chi_6 (f_6 - g_6) = \frac{M_6}{2}\left((\gamma - 1)(f_6 + g_6) + c_h \frac{s'_6}{c_p}\right). \tag{41}$$

Note that for the limiting case of low Mach numbers,

$$g_6 \approx f_6 - \frac{u_h}{2 c_p \chi} s'_6\,, \tag{42}$$

i.e. the reflected pressure signal g_6 is opposite in phase to the incoming entropy wave s'_6, provided that χ is a real number.

7. RESULTS

In the following, we discuss solutions of the system of Eqs. (33)-(41) for a premixed combustor as shown in Fig. 1. Model parameters were chosen to correspond, in order of magnitude, to those of premixed combustors of stationary gas turbines: operating pressure $p = 1.9 \times 10^6$ Pa, speed of sound $c_c = 550$ m/s and $c_h = 800$ m/s upstream and downstream of the flame, respectively, fuel injector loss coefficient $\zeta = 2$, Mach number at fuel injector $M_2 = 0.1$, mixing tube residence time $\tau = 1$ ms, combustor residence time 15 ms, and area ratio $\alpha = 0.3$.

For the flame forcing amplitude $\in$ of the driving term Eq. (23), we choose either $\in = 0.05$, i.e. we assume that 5% of the fuel injected is burned in an unsteady fashion, or we assume that the influence of coherent vortical structures on heat release fluctuations is negligibly small and set $\in = 0$. In the latter case, the linear stability of the system against self-excited instabilities can be investigated by searching for eigenfrequencies ω_m which satisfy the characteristic equation of the system or by using graphical methods derived from those commonly used in control theory.[16] In the present model, all transfer matrices are given analytically and it is most convenient to determine eigenfrequencies ω_m and corresponding eigenvectors by numerical root-finding procedures. The *stability* of the system is then determined by the imaginary part of the eigenfrequencies. With the sign convention used in this paper, a mode m is unstable if its imaginary part $\Im(\omega_m) < 0$. An indicator for stability more intuitive than the imaginary part of the eigenfrequencies is the *cycle increment* ς, defined as

$$\varsigma(\omega_m) \equiv \exp\left\{-\frac{\Im(\omega_m)}{2\pi\Re(\omega_m)}\right\} - 1 .$$

The cycle increment of an eigenmode equals the percentage by which the amplitude of an infinitesimal perturbation grows during one cycle of the oscillation; negative cycle increments indicate stability.

In the case of non-zero flame forcing, one can in general solve explicitly for the *response* of the system to the inhomogeneity. The amplitudes of fluctuations are in the linear case proportional to the forcing amplitude $\in$. In the following, using the equations presented in the previous section of this paper, the impact of entropy waves on both the stability and the response of the combustion system shown in Fig. 1 for the lowest plane and non-plane eigenmodes is explored. This is done by comparing results of stability analysis and response calculations, which consider entropy effects against corresponding analyses where the coupling of entropy waves with combustor acoustics has been neglected.

To obtain an order of magnitude estimate for the frequency of the fundamental mode of the combustor, one might argue as follows. In the limit of vanishing Mach numbers, the combustor should behave very roughly like a simple duct with an acoustically open end towards the burner plenum and a closed end towards the turbine inlet. With the values chosen for the speed of sound $c_h = 800$ m/s and combustor length $L_C = 0.5$ m, the eigenfrequency of the corresponding 1/4-wave mode is $f_0 \approx 400$ Hz. Indeed, Fig. 2, which shows eigenfrequencies obtained from stability analysis *without entropy* indicates that there is an eigenfrequency near 370 Hz, although the cycle increment $\varsigma \approx -0.86$ suggests that this mode is very strongly damped. The next eigenmode is found near 880 Hz, which although it is obviously not the frequency of the 3/4-wave mode is strongly unstable ($\varsigma \approx 0.34$).

It might at first seem surprising that the fundamental mode at 370 Hz is so strongly damped. However, the results of the stability analysis are corroborated by the corresponding response calculations, where — neglecting entropy effects — a resonance peak is found around 880 Hz (see Fig. 3), while the response of the acoustic system to the forcing is very weak near 370 Hz (not shown). The following explanation for the stability and the correspondingly weak response of the 1/4-wave mode may be offered. Fluctuations of heat release produce mainly, i.e. to 0-th order in Mach number M_4 — see Eq. (21) — fluctuations of velocity and only to 1st order in Mach number fluctuations in pressure. In other words, as far as thermo-acoustic phenomena are concerned, a perturbed heat source is primarily a "fluctuating source of volume". Near 370 Hz, the impedance $Z_4 \equiv p'_4/u'_4$ at the location of heat release (4) (see Fig. 1) is very small and the volume fluctuations produced by Q'_{Y_F} cannot couple effectively with the pressure field inside the combustor. The observation that the response calculation predicts $p'_4 \approx 0$ at 400 Hz, where the impedance $Z_4 = 0$, lends additional support to this argument. Furthermore, consider that the only dissipative loss mechanism in the system of Eqs. (33)-(41) is associated with the pressure loss between the burner plenum (1) and the location of fuel injection (2). For non-zero Mach number M_2, losses will be large if the fluctuations of velocity u'_2 at the fuel injector are large — and this is obviously the case for the 1/4-wave mode.

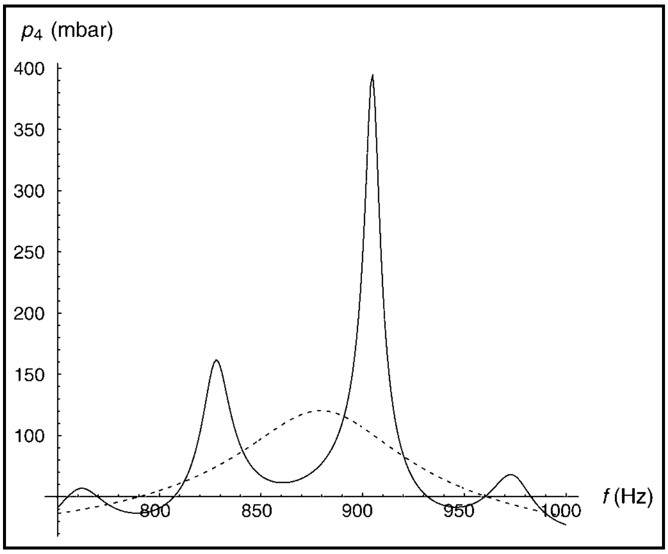

Figure 3. Amplitude of pressure fluctuations for plane waves ($k_\perp = 0$) at the flame front (Position 4). Response with (solid line) and without (dashed line) the coupling of entropy waves with the combustor acoustics. The frequency range in this plot covers the first resonance peak.

Let us now turn to the results obtained with entropy effects included. Figure 2 indicates that in this case a large number of additional eigenmodes appear, all of them stable. The frequency interval between these modes is approximately 62 Hz, which corresponds very nicely with the total time lag $\tau + \tau_C = 0.016$ s. A similar spacing in frequency is found for resonance peaks in the range 750-1000 Hz shown in Fig. 3. The deviations in amplitude between response calculations with and without consideration of entropy waves confirm that entropy waves can indeed couple constructively as well as destructively with the combustor, as was argued above. For the chosen parameter values, the primary resonance peak near 880 Hz is shifted to 905 Hz, while the amplitude more than triples if entropy effects are included in the response computation. Clearly, the entropy-induced pressure disturbances and the pressure field induced by vortex shedding and combustor acoustics can interfere constructively as well as destructively. Constructive interference occurs not only at one base-frequency corresponding to an *entropy mode* (approximately 76 Hz in the present case, see the first mode in Fig. 2), but also at higher frequencies. In that case, several "hot spots" are present in the combustor at any given moment, travelling downstream. In other words, the wavelength of the entropy disturbances is, in this situation, only a fraction of the combustor length.

The stability calculations, see again Fig. 2, show that the cycle increment of the mode near 370 Hz increases significantly due to entropy effects. This behaviour is explained as follows: although fluctuations of heat release in a region of low impedance Z cannot directly couple with the pressure field, they may very well generate entropy waves, which in turn generate pressure perturbations at the combustor exit. In general, this may result in self-excited combustion instability; for the present values of parameters all modes around 400 Hz display a negative cycle increment and are therefore stable. On the other hand, the mode near 880 Hz, which was found to be strongly unstable even without entropy effects, seems to be unaffected by entropy waves. An explanation for this result is also readily found. For a strongly unstable high frequency mode, the exponential factor in Eq. (40) will be very small, and consequently the amplitude of entropy perturbations at the exit s'_6 will be too small to generate a noticeable acoustic signal. We consider this effect to be to some extent an artifact of our simple model, where dissipative and dispersive effects have been largely neglected, which makes possible very large cycle increments. More realistic models of thermo-acoustic systems do in general not exhibit cycle increments larger than a few percent, and in such a situation the exponential factor in Eq. (40) would be of order unity. In other words, a change in the stability of a mode due to entropy effects could then be observed.

It is interesting to check whether the qualitative arguments involving relative phases are supported by the computational results. An analysis of the results of the response calculations reveals that the phase relations Eqs. (4) and (5) hold exactly. On the other hand, as was mentioned already, it is in general not easy to estimate the phase ϕ_f — see Eq. (6) — between velocity fluctuations u'_2 at the fuel injector and the pressure p'_6 at the combustor exit. For Helmholtz mode oscillations at low frequencies, p'_6 is approximately in phase with p'_2, and from Eq. (33) it follows that fluctuations of pressure and velocity are 180° out of phase. Indeed, $\phi_f \approx \pi$ was found for frequencies below 100 Hz. However, at higher frequencies such simple relationships cannot be established. For example, it is found that ϕ_f ranges from −1.2 to −1.4 radians in the frequency range 750 to 1000 Hz. Nevertheless, with ϕ_f determined from the solution of the inhomogeneous system of Eqs. (33)-(41), we find that indeed the interference of entropy and acoustic waves is constructive when the phases are more or less aligned with respect to each other, i.e. $-\pi/2 < \phi_{total} < \pi/2$, and destructive otherwise, see Fig. 4. On the other hand, it is obviously not true that amplification or suppression is strongest whenever ϕ_{total} is near π (or a multiple thereof). This is due to two reasons. Firstly, we have assumed idealised boundary conditions ($M \to 0$) when estimating the phase between convective and pressure waves at the combustor exit and the fuel injector. Secondly, there is additional interference between the heat release fluctuations Q'_Ω due to flame forcing (see Eq. (23)) and those due to equivalence ratio modulations (see Eq. (1)), which will influence the overall response to the excitation mechanism.

This concludes the presentation of results for plane waves, and we turn to a non-plane mode with $k_\perp$ set to 1.25 m^{-1}. For an industrial gas turbine, this could correspond to the first-order circumferential mode. Eigenfrequencies for non-plane waves with frequencies less than 400 Hz are shown in Fig. 5. Results obtained both with and without consideration of entropy waves are presented in this graph.

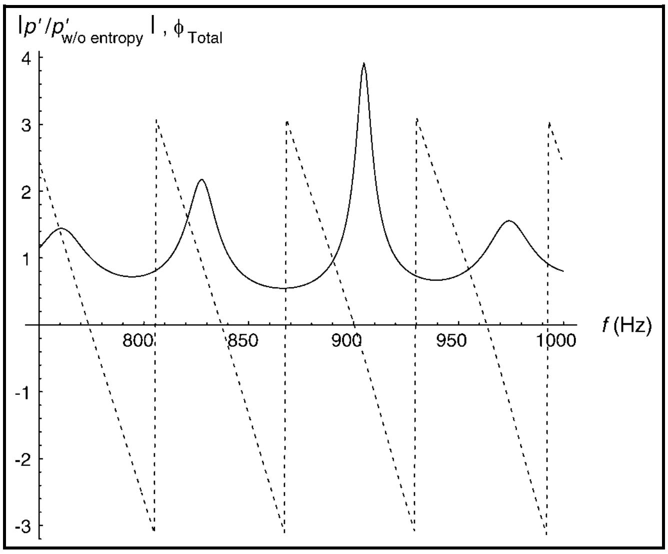

Figure 4. Solid line: Ratio of pressure amplitudes obtained with and without entropy wave effects. Dashed line: Total phase ϕ_{total}. Strong amplification of the response due to entropy wave effects is observed for $-\pi/2 < \phi_{total} < \pi/2$.

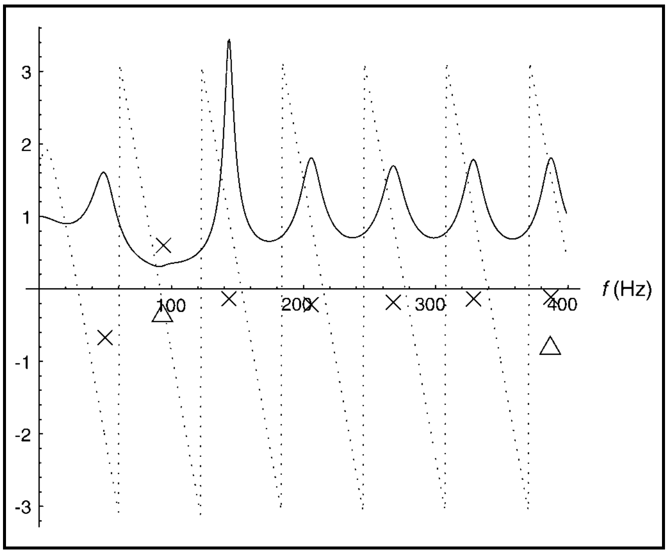

Figure 5. Cycle increment ς vs. frequency f of non-plane wave eigenmodes ($k_\perp = 1.25$ m^{-1}) in the range 0 to 400 Hz. Triangles — entropy waves neglected. “×“ — entropy wave effects included. Shown are also results from response computations for non-plane modes: the total relative phase ϕ_{total} (dotted line) and the ratio of pressure amplitudes obtained with and without entropy effects (solid line).

The cut-on frequency (the cut-on frequency $f_\perp$ is the eigenfrequency of a purely azimuthal wave with $k_x = 0$, therefore $(2\pi f_\perp/c)^2 = k_\perp^2$) of the combustor equals $f_\perp \approx 160$ Hz (with the value $c_h = 800$ m/s chosen for the speed of sound in the combustor). It follows that the first eigenmodes near 50, 94 and 143 Hz are all evanescent. Without consideration of entropy, only one propagating mode with $f = 387$ Hz is found in the frequency range considered. This mode is akin to a 1/4-wave mode, and as for the plane-wave case, the cycle increment ς is negative because of the small impedance at the location of heat release and high dissipative losses at the burner inlet. When entropy effects are included, we observe again that a large number of additional eigenmodes appear, with a frequency spacing approximately equal to the inverse of the total convective time lag $\tau + \tau_C$ for entropy waves. The two modes found without consideration of entropy both become less stable with the inclusion of entropy waves. However, only the evanescent mode near 94 Hz actually becomes unstable. For the chosen parameter values and in the selected frequency range, this is actually the only non-plane unstable mode found.

A comparison of the responses to flame forcing with and without entropy effects included is shown in Fig. 6. Again we find the characteristic modulation due to alternatingly constructive and destructive interference of the entropy waves with the combustor acoustics. With entropy waves considered, the strongest response is found near 143 Hz; while the peak found near 94 Hz without entropy effects is reduced to approximately one third of its original amplitude. This is a surprising result, considering that the corresponding eigenmode in the stability calculations was destabilised by the interference of entropy and acoustic waves. This unexpected behaviour is reflected also by the phase relationships (also

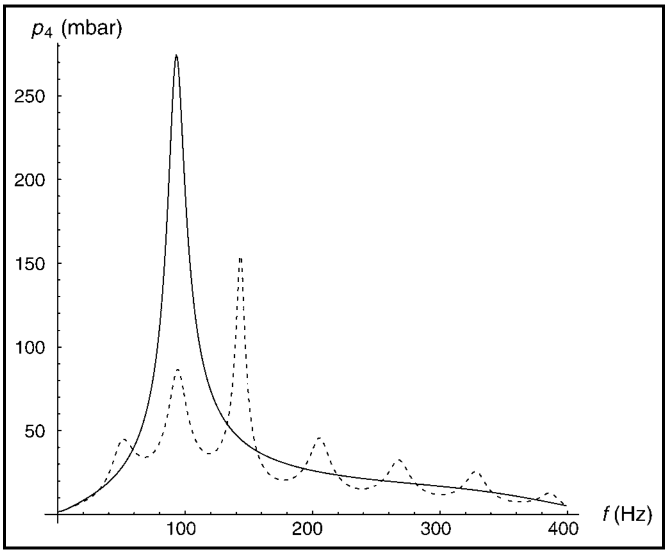

Figure 6. Amplitude of pressure fluctuations for non-plane waves ($k_\perp = 1.25$ m^{-1}) at the flame front (Position 4). Response with (solid line) and without (dashed line) the coupling of entropy waves with the combustor acoustics.

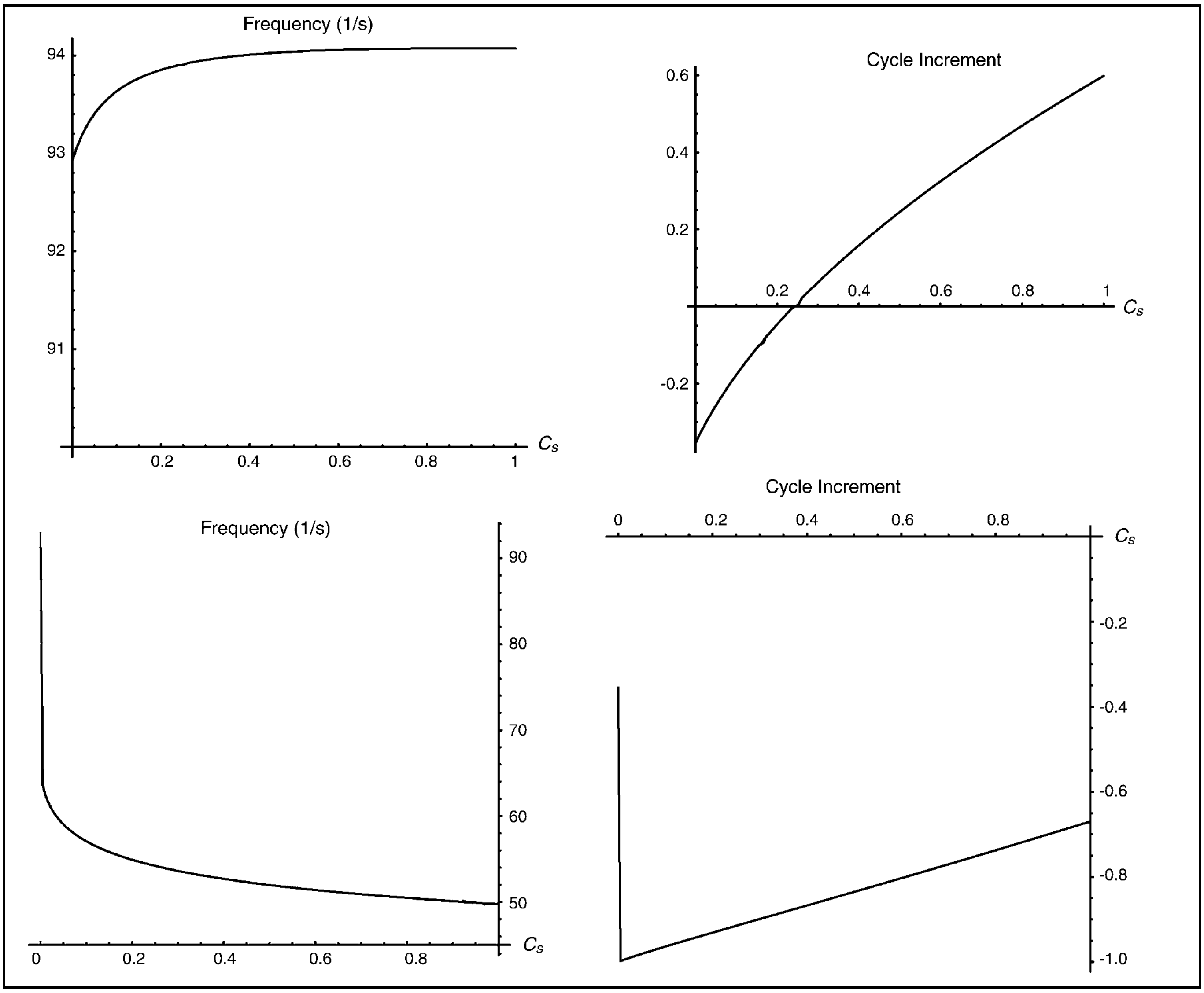

Figure 7. Changes in frequency (left) and cycle increment (right) of the first and second non-plane modes as entropy is "switched on" (i.e. C_S from $0 \rightarrow 1$).

shown in Fig. 5). For the propagating modes above 160 Hz, the frequencies of the eigenmodes correspond well with those of maximum amplification and are within those frequency bands where the total relative phase ϕ_{total} lies between $-\pi/2$ and $+\pi/2$. These relations are still satisfied for the mode at 143 Hz, but hold no longer true for the other evanescent modes near 93 and 50 Hz. For these two modes, the response is minimal when $\phi_{total} \approx 0$, and the eigenmode near 50 Hz shows $\phi_{total} \approx -2$. That is the phase relationship is now such that entropy waves interfere destructively with the combustor acoustics. The following explanation is proposed for these findings. The arguments based on the relative phase of entropy and pressure perturbations at the combustor exit are no longer adequate for evanescent waves with an imaginary axial wave number k_x. Indeed, for an evanescent wave, the factor χ appearing in Eq. (32) is a complex-valued number and Eq. (42) then shows that in this case the entropy and reflected pressure waves at the combustor exit are no longer opposite in phase. To predict the system behaviour, a detailed analysis of the interactions between the various interacting feedback mechanisms involved is required, which can only be provided by the solution of the full system of Eqs. (33) to (41).

To establish how, despite these complexities, the eigenmodes found with and without entropy correspond to each other, we show in Fig. 7 changes in eigenfrequency and cycle increment as entropy effects are gradually "switched on". To generate the graphs shown in the figure, a factor C_S has been introduced on the right hand side of Eq. (38) and varied from 0 to 1. Obviously, with $C_S = 0$, entropy waves are ignored, while with $C_S = 1$, entropy effects are included. With intermediate values of C_S, the eigenfrequencies can be tracked continuously through intermediate states. Although these intermediate states do not correspond in the present context to a physically realistic situation, the results illustrate nicely that frequency and cycle increment of the mode near 90 Hz — which is one of the eigenfrequencies found without entropy — increase continuously with both frequency and growth rates increasing from 93 to 94 Hz and from –0.33 to +0.60, respectively, as the coupling with entropy is switched on. The situation is different for the mode observed near 50 Hz when entropy effects are considered. Here discontinuous jumps in frequency and cycle increment to the values of the 90 Hz mode are observed at $C_S = 0$, suggesting that in this case a strong correspondence between modes observed with and without entropy cannot be established.

We conclude the presentation of results with Fig. 8, which shows how the frequency f and the cycle increment ς of the 90 Hz eigenmode mentioned previously change with combustor residence time τ_C. Note that when generating the data shown in this plot, only the parameter τ_C appearing in Eq. (39) has been changed, no corresponding changes in combustor length L_C or flow velocity u_h have been implemented. The qualitative analysis, based on relative phases discussed above, suggests that the interaction between entropy and acoustic waves should react in a very sensitive manner to changes of the combustor residence time. This is confirmed by the numerical results. When τ_C is varied between 10 to 35 ms, the eigenfrequency of the mode considered decreases from 120 to 50 Hz, while the cycle increment first increases to a value around 0.6 and then decreases to –0.4. This indicates that the mode is very strongly damped for this value of combustor residence time.

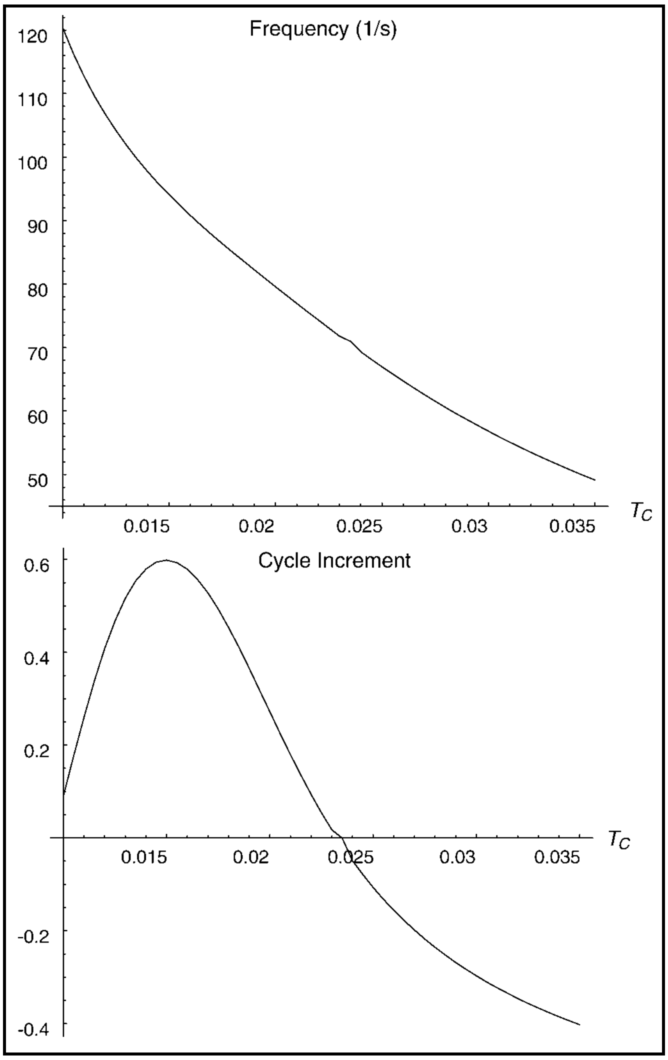

Figure 8. Changes in frequency (top) and cycle increment (bottom) of the second non-plane modes ("90 Hz mode" for τ_C = 15 ms) as combustor residence time τ_C varies from 10 to 35 ms.

8. DISCUSSION AND CONCLUSIONS

It has been argued that the coupling between entropy waves and combustor acoustics at a choked exit can strongly influence the acoustic response of a combustor, even at frequencies that are higher than those associated with combustor residence times or — more generally speaking — convective time lags. Depending on the relative phase between entropy fluctuations and combustor acoustics, entropy effects can enhance as well as reduce the susceptibility of a combustion system to thermo-acoustic oscillations.

Some of the phase relations involved in this interaction of acoustic and convective waves may be estimated in a straightforward manner from consideration of the various time lags and phase jumps involved in the propagation and interaction of acoustic and convective waves. At least for plane waves in the limit of low frequencies, such considerations allow a first estimate of the impact of entropy waves on combustion stability. However, in general it is not possible to determine all relevant phase relations without a more detailed and quantitative analysis of the wave propagation processes in the combustion system. Also, phase relations alone do not provide an answer to the question, whether the coupling of entropy waves and combustor acoustics is strong enough to significantly influence the combustion system's thermoacoustic behaviour. Therefore, an idealised, quasi-1D, quantitative model of a premixed combustor has been formulated, which allows one to compute the response to acoustic driving by unsteady heat release as well as determine the stability against self-excited combustion instabilities.

We emphasise that the model presented in this work was designed to illustrate, clarify and formulate mathematically ideas concerning interference of convective and acoustic waves. To this purpose, we have neglected phenomena like dispersion of fuel inhomogeneities or "hot spots" by turbulent diffusion or convective transport, distributed heat release or distributed fuel injection, vortical velocities, etc., which would make the presentation much more complex and harder to follow. Also, dissipative losses of acoustic energy, e.g. at the burner dump plane or the liner walls, have not been considered. As one result of these simplifications, growth rates have been observed which must be judged to be unrealistically large. Nevertheless, it is expected that the model produces physically reasonable results and predicts realistic trends.

Response calculations and stability analyses carried out with the model confirm that indeed the coupling between entropy and acoustic perturbations at the exit nozzle can be strong enough to significantly influence the frequencies and growth rates of the system's eigenmodes. Also, once entropy effects are introduced, a large number of additional modes appear, which have no counterpart in a model formulation which ignores such effects. These additional modes appear in general to be stable with respect to self-excited combustion instabilities. A detailed analysis of solutions obtained with the model confirms that simple arguments, based on relative phases between entropy and acoustic waves, are roughly fulfilled. Deviations from this kind of reasoning can be explained with the idealised treatment of boundary conditions and the interference of heat release fluctuations due to equivalence ratio fluctuations with the forcing term. Moreover, it is generally true that modes which respond strongly to driving by an inhomogeneity are also prone to self-excited instabilities and vice versa. An exception to this rule has been observed for evanescent non-plane modes.

Since the first presentation of the ideas presented in this paper,[12,13] Sattelmayer[17] has carried out a detailed stability analysis of a model combustion system similar to the one presented here, with the important difference that the dispersion of fuel equivalence ratio and entropy modulations by the mean flow are taken into account. Supporting some of our arguments, Sattelmayer emphasises the importance of relative phases and also points out that the superposition of convective and acoustic waves can in principle stabilise as well as destabilise a combustion system. It is concluded that for realistic thermo-acoustic analysis, the dispersion of convective waves must be taken into account while it is not mandatory to incorporate entropy waves in the stability analysis of a gas turbine combustor.

We certainly agree that the dispersion of convective waves must be treated properly when a realistic analysis of a thermo-acoustic system is attempted. We also concede that for high frequency modes, say the 880 Hz plane mode displayed in Figs. 2 and 3, the dispersion of the entropy waves is, for realistic mean flow distributions, in all likelihood so strong that the effect of entropy waves on combustion noise or instability is very weak. On the other hand one should keep in mind that Sattelmayer[17] has considered only plane waves. For non-plane waves the axial wave number k_x is much smaller than for plane waves, which means that dispersive effects will be correspondingly weaker. Also, Fig. 8 shows very clearly that the growth rates achieved depend very strongly on system parameters which control the relative phases. It is not clear from the results presented whether the parameters used by Sattelmayer[17] were adjusted for optimal phase alignment. In other words, with a different set of parameters, a strong response to entropy waves may be observed even when dispersion of convective waves is taken into account. We therefore affirm our original conclusion[12,13] that the thermo-acoustic analysis of a combustor with a choked exit should in general include the effects of entropy waves, and that — as the title of this paper suggests — careful tuning of convective time lags against combustor resonance frequencies can aid in suppressing self-excited combustion instabilities.

Stability criteria for premixed combustion based on the relative phases between convective and acoustic waves as they have been discussed in this work have also been independently developed and used by other authors, see e.g. Janus and Richards[18] or Lieuwen et al.[19] Such criteria are undoubtedly very useful for the conceptual development and analysis of feedback loops involving convective and acoustic waves. Unfortunately, experience shows — see e.g. Fig. 10 in Janus and Richards[18] or Fig. 8 in Lieuwen et al.[19] — that stability criteria based solely on the relative phases of pressure and fuel equivalence ratio fluctuations are at best a necessary but not sufficient criterion for instability to occur. In other words, quite frequently an instability is not observed in spite of matching phases. Dissipative losses of acoustic energy[19] and dispersion of convective waves can be held responsible for these observations. However, it has also been observed that an instability can occur in regions where the phase-based stability criteria suggest otherwise.[19]

The arguments developed in this work and the results obtained with the quantitative, detailed model point to other possible explanations for these observations.

1) interference of acoustic and entropy waves at a choked cross section,
2) errors in the prediction of relative phases due to the idealised treatment of boundary conditions or the neglect of phase jumps at area discontinuities,
3) interaction of two (or more) mechanisms of heat release modulation, e.g. fluctuations of equivalence ratio and coherent structures, but also flame front dynamics or chemical-kinetic time lags, and
4) small impedance at the location of the heat release.

Only detailed analysis of the test rigs, including a realistic treatment of convective wave propagation and dispersion, of dissipative losses, of boundary conditions and of the flame front dynamics can show which of the effects mentioned is dominant in controlling thermo-acoustic instability in premixed combustion systems.

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This paper is dedicated to the memory of our dear friend and colleague Jakob J. Keller (1950-1999). We gratefully acknowledge discussions with Alexander Ni.

REFERENCES

[1] Tsien, H.S. The Transfer Function of Rocket Nozzles, *J. of the American Rocket Society*, **22**, 139-143, (1952).

[2] Marble, F.E., and Candel, S.M. Acoustic Disturbance from Gas Non-Uniformities Convected through a Nozzle, *J. of Sound and Vibration*, **55**, 225-243, (1977).

[3] Toong, T.Y. Basic Instability Mechanisms in Chemically Reacting Subsonic and Supersonic Flows, Technical Report AFOSR-78-3662, (1982).

[4] Keller, J.J., Egli, W., and Hellat, J. Thermally Induced Low-frequency Oscillations, *Zeitschrift für angewandte Mathematik und Physik*, **3** (2), 250-274, (1985).

[5] Culick, F.E.C. Combustion Instabilities in Propulsion Systems, *Combustion Instabilities Driven by Thermo-Chemical Acoustic Sources*, NCA-Vol. 4, HTD-Vol.128, ASME, 33-52, (1989).

[6] Macquisten, M.A., and Dowling, A.P. Low-frequency Combustion Oscillations in a Model Afterburner, *Combust. and Flame*, **94**, 253-264, (1993).

[7] Keller, J.J. Thermoacoustic Oscillations in Combustion Chambers of Gas Turbines, *AIAA Journal*, **33** (12), 2280-2287, (1995).

[8] Dowling, A.P., and Hubbard, S. Acoustic Instabilities in Premix Burners, *4th AIAA/CEAS Aeroacoustics Conference*, Toulouse, (1998).

[9] Zhu, M., Dowling, A.P., and Bray, K.N.C. Self-excited Oscillations in Combustors with Spray Atomisers, *Int'l Gas Turbine and Aeroengine Congress & Exposition*, ASME, **2000-GT-108**, (2000).

[10] Humphrey, J.W., and Culick, F.E.C. Pressure Oscillations and Acoustic-Entropy Interactions in Ramjet Combustion Chambers, *23rd Joint Propulsion Conference*, San Diego, AIAA/SAE/ASME/ASEE, (1987).

[11] Chu, B.T. On the Generation of Pressure Waves at a Plane Flame Front, *4th Symposium (International) on Combustion*, 603-612, (1953).

[12] Polifke, W., Paschereit, C.O., and Döbbeling, K. Coupling of Acoustic and Entropy Fluctuations in a Premixed Combustor with Choked Exit. *Work in Progess Poster*, *27th Symposium (International) on Combustion*, Boulder, CO, (1998).

[13] Polifke, W., Paschereit, C.O., and Döbbeling, K. Suppression of Combustion Instabilities through Destructive Interference of Acoustic and Entropy Waves, *Proceedings of the 6th Int. Congr. on Sound and Vibration*, Copenhagen, Denmark, (1999).

[14] Paschereit, C.O., Schuermans, B., Polifke, W., and Mattson, O. Measurement of Transfer Matrices and Source Terms of Premixed Flames, *Int'l Gas Turbine and Aeroengine Congress & Exposition*, ASME, Indianapolis, Indiana, USA, (1999).

[15] Munjal, M.L. *Acoustics of Ducts and Mufflers*, John Wiley & Sons, (1986).

[16] Polifke, W., Paschereit, C.O., and Sattelmayer, T. A Universally Applicable Stability Criterion for Complex Thermoacoustic Systems, *18. Deutsch-Niederländischer Flammentag*, VDI Bericht, Delft, NL, **1313**, 455-460, (1997).

[17] Sattelmayer, T. Influence of the Combustor Aerodynamics on Combustion Instabilities from Equivalence Ratio Fluctuations, *Int'l Gas Turbine and Aeroengine Congress & Exposition*, ASME, Munich, Germany, **2000-GT-82**, (2000).

[18] Janus, M.C., and Richards, G.A. Characterization of Oscillations During Premix Gas Turbine Combustion, *Int'l Gas Turbine and Aeroengine Congress & Exposition*, ASME, Orlando, FL, **97-GT-244**, (1997).

[19] Lieuwen, T., Torres, H., Johnson, C., and Zinn, B.T. A Mechanism of Combustion Instability in Lean Premixed Gas Turbine Combustors, *Int'l Gas Turbine and Aeroengine Congress & Exposition*, ASME, Indianapolis, IN, **99-GT-3**, (1999).