Inverse and Reciprocity Methods for Machinery Noise Source Characterization and Sound Path Quantification Part 2: Transmission Paths

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In this article and in a foregoing companion article some novel approaches to the characterization of the noise source strength of machinery and to the ranking of transmission paths are reviewed. They form an addition to the more conventional approaches of the analysis of noise control problems in terms of source-transmission path-receiver schemes.
In the first article source strength descriptors have been defined in terms of equivalent fictitious elementary sources, such as acoustical monopoles and mechanical point forces. This second article presents examples, which illustrate how some of these unconventional source strength descriptors have been exploited for sound transmission path quantification. These concern applications in ships and road vehicles. In all cases it is the combination with experimental reciprocity techniques, that makes the use of the elementary substitution sources as source models very powerful for transmission path ranking. Therefore, an appendix on reciprocity relations in structural-acoustics is added to this article, including a brief bibliography on that subject.

1. INTRODUCTION

This article and the foregoing companion article [1] are concerned with the experimental analysis of machinery noise control problems. Although the general approach, in terms of a source - transmission path - receiver scheme, is well-known, this is not the case with the manner in which source modelling is handled.

The noise control problems of interest here, are dealing with a multitude of simultaneous sources or with a single noise source like a machine, with a multitude of partial sources and 'parallel' transmission paths. The analysis models which are considered are limited to systems with supposedly linear behaviour, i.e. to systems for which output responses may be modelled as a linear superposition of all contributions of the partial sources and transmission paths. Then in loose mathematical terms, for example, the following model equation applies:
- *m partial sources:*

$$O_k = \sum_{j=1}^{m} TF_{j,k} \times I_j . \qquad (1)$$

This type of analysis requires a system modelling in terms of partial inputs I_j, which genuinely characterize the partial sources themselves and of output-input ratios O_k/I_j. These latter are transfer functions $TF_{j,k}$, which characterize the transmission paths.

In the first part of this article two examples are presented, which illustrate the use of the 'uncorrelated equivalent monopoles' method [1] for the analysis of airborne sound transmission. The first example is concerned with the airborne sound transmission from a shipboard diesel generator set to underwater. The second example is concerned with the evaluation of the effectiveness of tightly fitting engine encapsulations in heavy road vehicles.

In the second part of this article, application of the 'equivalent correlated point forces' method [1] for the analysis of structure-borne sound transmission will be illustrated for two examples of 'compact' sources. One is a hydraulic pump in a refuse vehicle and one is an electromechanical drive in a frame of a copier machine. A third application illustrates how this method of fictitious equivalent forces can be applied part way along a transmission path. This example concerns the sound transmission to underwater from the main gearbox in a ship via the flanking transmission path of a propeller shaft bearing.

2. METHOD OF UNCORRELATED EQUIVALENT MONOPOLES

2.1 Experiments on a shipboard diesel engine

In naval ships noisy machines are resiliently mounted to reduce the underwater sound radiation. However, the effectiveness of the vibration isolation may be spoiled to an unwanted degree by the 'flanking' airborne sound transmission. This might require costly extra noise reduction measures, like an enclosure. Therefore, the reliable analysis of the airborne sound transmission to the underwater in available ships is of great practical interest. The obvious approach of using loudspeakers as substitution sources is impractical in ships fo r two reasons. Usually the machine dimensions are large and the engine rooms are not reverberant. This would require a large number of loudspeakers to reproduce the original sound field with sufficient accuracy. The other problem is that large loudspeaker boxes are needed to exceed the background noise levels in the water. However, ships' engine rooms are usually not accessible with large loudspeakers. Therefore, for an auxiliary diesel engine on a frigate, the method of uncorrelated equivalent monopoles [1] was applied to quantify the airborne sound transmission to underwater.

For the analysis the following variant of Eq. (1) has been used:

$$p^2_{w,\,airborne} = \sum_{j=1}^{6} p^2_w(j) = \sum_{j=1}^{6} T_{j,w} \times Q^2_{eq}(j) \quad . \tag{2}$$

The mean square underwater sound pressure caused by airborne sound transmission was analysed as a summation of the partial contributions of six sub-areas on the engine surface. The equivalent mean square volume velocities $Q^2_{eq}(j)$ of these sub-areas have been determined from sound intensities measured around the running engine [1]. The transfer functions were measured in a harbour, with the engine switched off and using reciprocity experiments. During these reciprocity measurement the sound was generated with an electrodynamical underwater sound source with volume velocity Q'^2_w, see Fig. 1. For each of the six sub-areas on the engine, the mean square pressure p'^2_i was measured against the engine surface at $n(j)$ microphone positions and the transfer functions were averaged according to

$$T_{j,w} = \frac{1}{n(j)} \sum_{i=1}^{n(j)} \left[\frac{p'^2_i}{Q'^2_w} \right] . \tag{3}$$

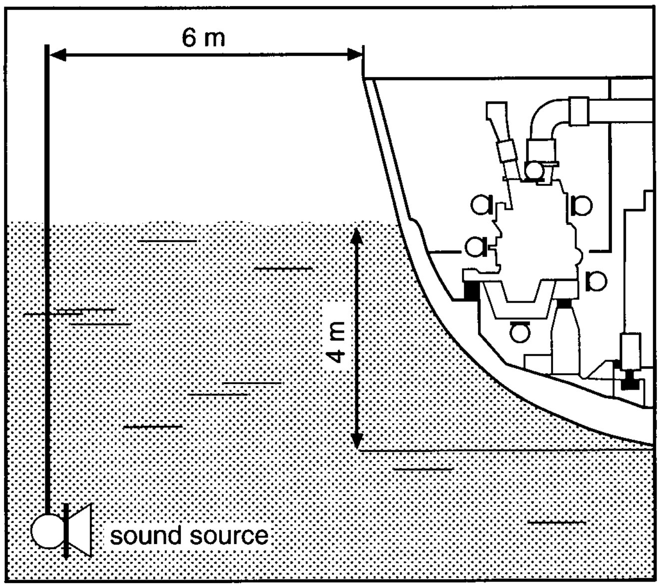

Figure 1. Reciprocal measurement of transfer functions according to Eq. (3) on frigate.

Figure 2 shows reciprocally measured transfer functions from the top surface and from the engine raft bottom. Notice the large differences in transmission attenuation.

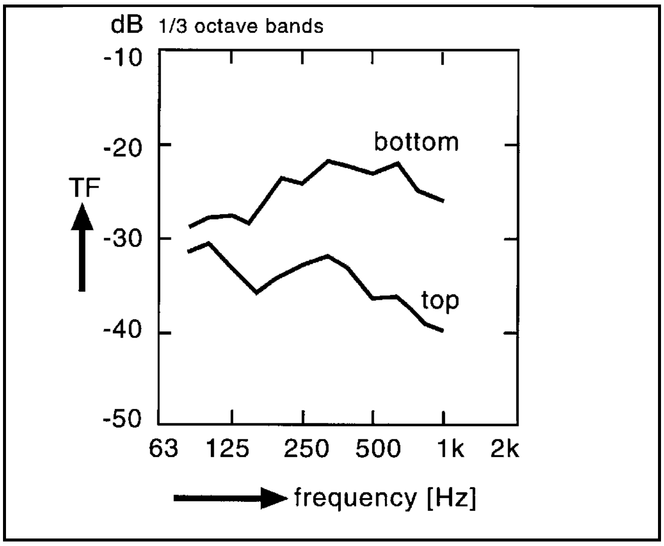

Figure 2. Transfer functions to underwater from different source areas on diesel engine.

According to Eq. (2), the multiplication of such transfer functions with the corresponding equivalent volume velocities, gives an estimation of the underwater sound due to airborne sound transmission for the individual sub-areas and for the total engine. These underwater sound pressure levels are not shown here for reasons of confidentiality. Nevertheless, in addition to earlier reported result of validation studies[1], some useful information on the feasibility and 'robustness' of this analysis method can be presented here.

The sound intensity measurements, which were used to derive the equivalent monopole strengths, were performed twice, with a period of eleven days in between. Typically the one-third octave levels of the six partial sound powers of the sub-areas were repeatable within 1.5 dB.

The transfer function measurements with the underwater sound source in the harbour have been performed for 60 microphone positions against the engine surface. The calculations according to Eqs. (2) and (3) were made using these 60 transfer functions as well as a subset using 26 instead of 60 microphone positions. The predicted one-third octave band underwater sound pressure levels did not differ more than 1 dB.

From these and other results the conclusion was, that this reciprocity method for path analysis is practically feasible. It provides the partial contribution of airborne sound transmission compared to the totally radiated underwater sound due to the diesel engine. Moreover, it provides rather detailed information on the relative importance of the various parts of the machine surface and of the corresponding transfer functions. This information can aid the cost-effective realization of design improvements.

2.2 Insertion loss experiments on engine encapsulations

In a recently finished EU-sponsored project (PIANO) a consortium of manufacturers, acoustical firms, research institutes and universities has investigated new methods that enable faster, cheaper and more accurate optimization of noise reduction measures for heavy road vehicles. One of the tasks was to validate the application of the same reciprocity technique as discussed in section 2.1, but now for the analysis of airborne sound transmission of engine noise to pass-by noise microphones. For example, there is great interest in a quick and reliable method for the acoustical evaluation of engine encapsulations and shieldings. A good measure for the performance of the encapsulation effectiveness is the so-called insertion loss or the change in insertion loss, i.e.

$$IL_p = L_p(\text{without encapsulation}) - L_p(\text{with encapsulation}) \tag{4}$$

or

$$\Delta(IL_p) = L_p(\text{encapsulation } 2) - L_p(\text{encapsulation } 1) \quad . \tag{5}$$

The sound pressure levels in Eqs. (4) and (5) are the partial contributions at the pass-by microphone positions due to airborne sound transmission from the engine. These sound pressure levels can be analysed similarly as in Eq. (2), using

$$p^2_{mic}(airb., eng.) = \sum_{j=1}^{m} p^2_{mic}(j) = \sum_{j=1}^{m} T_{j,mic} \times Q^2_{eq}(j). \tag{6}$$

The analysis results according to Eqs. (4)-(6) do not only provide insertion losses. They reveal additionally, for example, which parts of the engine give important contributions to the pass-by noise and which parts of an encapsulation need further improvement.

The practical advantage of this analysis procedure can be explained as follows. It is assumed that the equivalent volume velocities of engine parts remain unaltered. Therefore, the source terms in Eq. (6) need to be determined only once. They are needed as 'weighting factors'. However, in case of encapsulation or shielding modifications, only the transfer function measurements have to be repeated, using reciprocal experiments with the engine switched off.

As discussed elsewhere[1], compared with the shipboard application in 2.1, the essential difference in this road vehicle application is that the source strength determination cannot be performed in situ, i.e. with the engine installed in a small space inside the vehicle. Instead these measurements have to be performed on a test rig, on which the radiation resistance of the engine may differ significantly from that in the vehicle. Therefore, the critical assumption for this vehicle application is the invariance of the equivalent volume velocities of engine parts for drastic changes in radiation loads. In addition to previously discussed evidence for the validity of this assumption[1], here some experimental evidence is added. These data were obtained from laboratory experiments, which were performed prior to the industrial implementation of the method, to increase confidence in it and to investigate certain practical aspects.

In Table 1 sound pressure levels and insertion loss values are presented for laboratory models.

The data have been obtained from measurements in a semi-anechoic room [2]. These measurements were performed on an engine simulator without enclosure as well as inside a full enclosure and inside an enclosure without bottom plate respectively. Vehicle engine sound was simulated for a steady state condition. In the post-processing the source spectrum of the bare engine simulator was made equal to that of a representative truck engine on a test rig.

To predict the sound pressure level for the situation without enclosure, equivalent volume velocities were determined on the bare engine simulator using intensity measurements [1]. In addition the transfer functions to six sub-areas of the source were determined reciprocally, in the same way as in Eq. (3). In the predictions for the enclosure variants, the source strengths measured on the bare source were combined with reciprocally measured transfer data for microphones inside the enclosures and put against the engine simulator surface.

Table 1. Wide band sound pressure levels (A-weighted) and insertion losses at a few meters distance from the engine simulator and after conversion to a representative diesel engine spectrum.

	Side position		Front position	
	measured	predicted	measured	predicted
L_A without enclosure, dB(A)	117	117	115	115
L_A with full enclosure, dB(A)	91	91	91	90
L_A for enclosure without bottom plate, dB(A)	112	111	112	112
Insertion loss: full enclosure, dB	26	26	24	25
Insertion loss: partial enclosure, dB	5	6	3	3

From Table 1 a very good agreement is seen between directly measured results and calculations with Eqs. (6) and (4). This was also the case for other experiments, in which local apertures were made in the enclosure and where additional local sources were attached on the engine. In all cases the predictions based on Eq. (6) appeared rather insensitive to the precise number and location of substitution monopoles. Therefore, at the start of the industrial implementation, there was good reason to expect that the reciprocity experiments combined with the novel source descriptor, form a valid and very practical tool in studying shielding and encapsulation variants.

Some information on the industrial application of this method has been reported elsewhere [3,4].

3. METHOD OF CORRELATED POINT FORCES

In another recently finished EU-sponsored project (EQUIP), a consortium of manufacturers, research institutes and universities has worked on the methodology and tools for developing quiet industrial products. One of the tasks was to develop further experimental methods for the characterization of structure-borne sound sources. For this purpose the application of the 'correlated equivalent point forces' method have been studied on an electromechanical copier drive and on an hydraulic pump. The method itself and some results from these studies have been discussed in the earlier companion paper[1]. Here, some additional results are discussed, in order to illustrate the practical value of this method in relation to diagnostic analysis and design optimization.

In modelling sources of structure-borne sound with the 'correlated equivalent point forces' method, the internal excitation of mechanical components is modelled with a set of fictitious external forces upon the source body. Together, these forces are called 'equivalent', if they produce the same source body vibrations as the internal excitation mechanism. For the determination of these equivalent pseudo-forces, a so-called inverse method is used in three steps. With the source in operation, accelerations are measured at a number of positions and in different directions. With the source switched off, accelerances are measured between the acceleration response positions and the positions of the fictitious forces. Finally, in an analytical step the pseudo-forces are calculated, which, when acting together, would lead to the best reproduction (in a least squares sense) of the accelerations measured with the source in operation.

For an hydraulic pump it was shown earlier [1], that the accelerations caused by it on a frame of a refuse vehicle could be 'predicted' quite accurate on the basis of a source transfer path model similar to that in Eq. (1). This appeared to be true, both for equivalent forces determined on the pump installed in the vehicle and on the pump installed in a test rig of the pump manufacturer.

The variant of Eq. (1), which was used for this application, is:

$$\boldsymbol{a} = \boldsymbol{H}\boldsymbol{F}_{pseudo}, \tag{7}$$

where **a** denotes the vector of accelerations on the vehicle frame, $\boldsymbol{F}_{pseudo}$ the vector of equivalent forces and **H** the matrix of frequency response functions which connect the individual forces and accelerations. Again in practice it will be often convenient, from a viewpoint of accessibility, to measure the elements of **H** reciprocally, i.e. by interchanging force and acceleration positions.

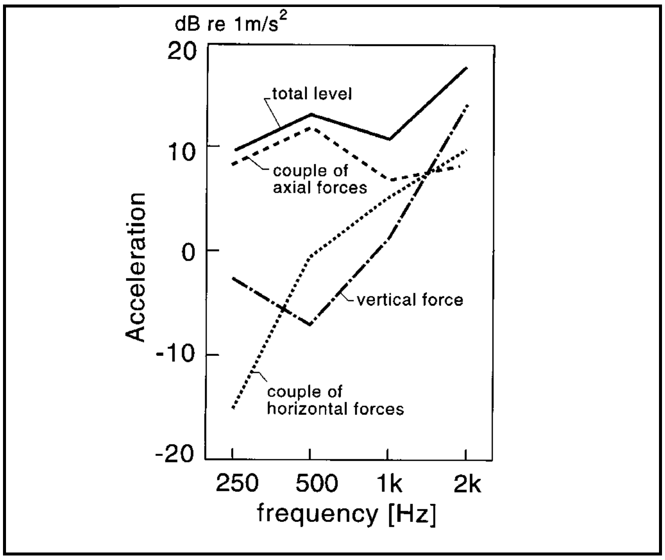

Figure 3. Calculated acceleration levels on refuse vehicle frame as caused by pseudo-forces on hydraulic pump.

Figure 3 shows levels of mean square accelerations averaged over 8 positions on the frame of the refuse vehicle. These were calculated using Eq. (7) and using the set of pseudo-forces obtained from experiments in the test rig of the pump manufacturer. However, in addition to the total responses, figure 3 shows also the most important partial contributions from individual 'orthogonal' forces and 'force-couples'. The contribution due to the 'couple of axial forces', which was constructed from the difference of two axially directed pseudo-forces on the back of the pump, is seen to be dominant at low frequencies. The strong excitation by this couple, seems to be related with the internal multi-piston excitation. However, not surprisingly, as will be shown next, it is not always the dominant excitation component, which will dominate the noise production.

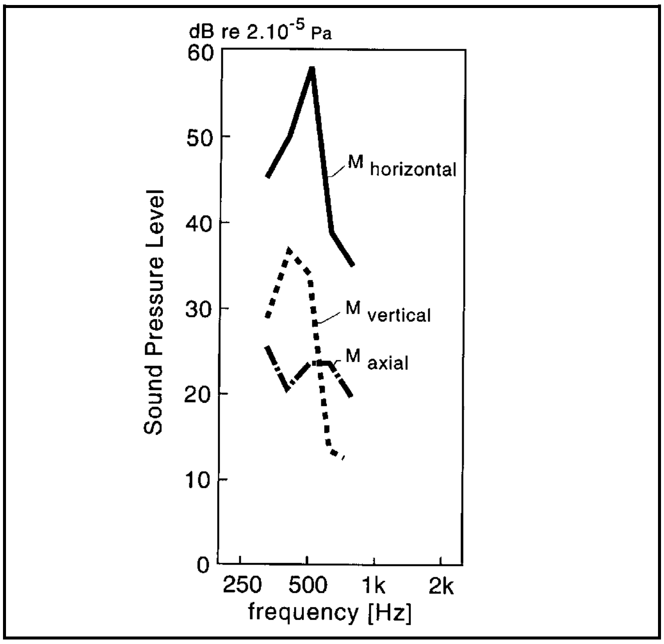

Figure 4. Calculated sound pressure levels as caused by pseudo-couples on a drive in a copier frame.

Figure 4 shows partial contributions to the radiated sound for an electromechanical drive, when mounted on a frame of a copier machine [1,5]. These were obtained from an analysis similar to that in Eq. (7), using

$$\boldsymbol{p} = \boldsymbol{H}\boldsymbol{F}_{pseudo}\ . \tag{8}$$

Now **H** denotes a matrix of frequency response functions between the pseudo-forces or pseudo-couples and sound pressures at positions around the copier frame. The figure shows that the partial contribution from the couple about the horizontal axis parallel to the frame is by far predominant. In contrast to the hydraulic pump, in this case the cause is not a particularly strong excitation component in the drive, but a strong transmission for this particular excitation. For designers this is important information, because in this case a rather simple modification of the frame structure may reduce the radiated sound considerably. Reciprocity experiments form an elegant and rather simple tool to discover such "weaknesses" in the transmission. However, then a passive source or a representative 'dummy' needs to be evaluated as part of the transmission system. And in addition, appropriate pseudo-forces need to be known.

4. METHOD OF CORRELATED EQUIVALENT POINT FORCES WITHIN TRANSMISSION PATHS

In the foregoing section and in the foregoing companion article[1] the modelling with correlated equivalent point forces was applied for forces upon a source structure and for the analysis of transmission paths which include this source. However, variants of the same technique have been applied with the fictitious equivalent forces acting not on a source structure, but part way along a transmission path. Bowen and Lyon[6] have analysed the structure-borne sound transmission via the mounts of a cooling fan. The excitation via the mounts of the carrier structure was modelled with four equivalent forces. Verheij et al.[7] report the analysis of transmission from a truck chassis into the cabin via four vibration isolators. Here the transmission via the isolators was modelled with twelve forces, i.e. three perpendicular forces for each isolator. They also describe laboratory tests in which the practicability of the technique is tested for sound path quantification in ships. Results are presented for transmission via a propeller shaft bearing and via a liquid-filled pipe system. Others have reported applying the technique for sound path analysis in cars [8-10].

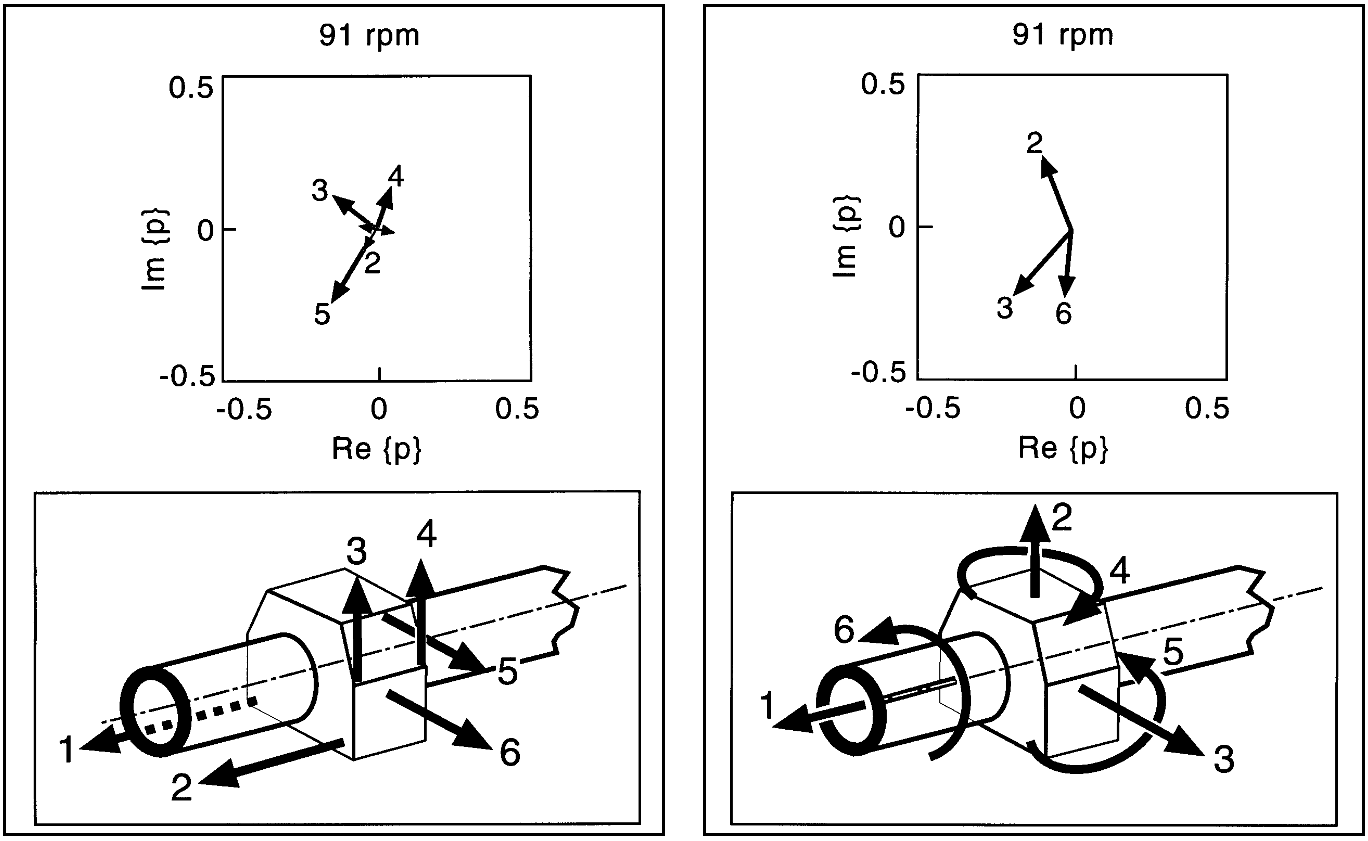

Figure 5. Calculated underwater sound pressure of a gearbox tone for flanking transmission via a propeller shaft bearing. Top left: for equivalent forces at measurement positions (see bottom left). Top right: for transformed set of equivalent forces and force-couples (see bottom right).

In recent years the author and his colleagues have applied this technique extensively and successfully in shipboard applications. Topics were the multi-point and multi-directional transmission via a flexible foundation structure to underwater, the flanking transmission of gearbox noise via the propeller shaft and a shaft bearing to underwater, and the transmission via liquid-filled pipes and power cables to underwater. All these applications have in common that path analysis methods which require dismantling of systems are virtually impracticable. The elegance of the equivalent forces method is that the inversion procedure is not only able to estimate genuine internal forces at the interconnections between substructures, but also fictious external forces or 'pseudo-forces', which together reproduce the original vibration field on the sound path downstream of these equivalent forces. When the sound propagation to the far field due to this equivalent forces excitation is predominantly determined via this downstream transmission path, the path contribution can be estimated using Eq. (7) or (8).

As an example, figure 5 shows some data of the application of this method on a propeller shaft bearing in a frigate. The analysis was concerned with flanking sound transmission from a main gearbox to underwater via the shaft and the bearing structure. The three steps in the path analysis procedure were executed as follows. At sea the accelerations were measured at twelve positions on the bearing structure and on its foundation at various running speeds. The accelerance matrix was measured with the ship in a harbour. Six forces were applied on the bearing structure; see Fig. 5 at bottom left. Transfer functions for the equivalent forces to a farfield underwater position were determined both in a harbour and on a sound range.

Again for reasons of confidentiality, detailed results of the path ranking cannot be shown. However, the results presented in Fig. 5 illustrate a particular aspect of the analysis. The figure shows the calculated sound pressures at one of the running speeds. The presentation is in the complex plane and shows the partial contributions of the individual equivalent forces. On the left hand side the result is shown for the set of equivalent forces at their 'actual' points of excitation. On the right hand side the result is shown after linear transformation into an 'orthogonal' set of forces and force-couples [1]. After this transformation it is clearly seen that tranverse forces, probably caused by the bending vibrations of the propeller shaft, are the main cause of the flanking transmission to underwater via the bearing structure.

Similar analysis at other speeds and for another gearbox tone, showed consistent results both with respect to the path ranking and with respect to the predominance of transverse excitation of the bearing structure.

In this brief overview of applications a lot of important practical aspects of the method cannot be treated. An important aspect is the modelling, i.e. the choice of force and response positions. Other aspects are signal processing, dealing with responses in a multi-source environment, dealing with contributions to the responses used in the inversion by more than one transmission path and dealing with inaccuracy of responses used in the inversion and with weighting of these responses. Concerning the modelling problem, recently the application of the equivalent forces method on sound transmission path ranking for liquid-filled pipes was investigated in a numerical simulation study[11]. It is shown that both the transmission through the pipe wall and that through the liquid can be modelled sufficiently accurately by using only structural excitation forces.

5. CONCLUSION

This paper and the foregoing companion paper have shown how experimental sound source characterization and sound path quantification can often be facilitated by modelling the original excitations through fictitious excitations using acoustical monopoles or mechanical point forces. In combination with reciprocity techniques, these methods form powerful additions to the tools in noise control engineering.

6. ACKNOWLEDGEMENTS

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APPENDIX: POINT-TO-POINT RECIPROCITY RELATIONS

In the field of structural acoustics there are many applications of reciprocity, both in theoretical treatments and in experimental work. Few of them have been used in the foregoing article.

Loosely speaking, the principle of reciprocity states that the response of a passive linear system to an external input, is unchanged if the points of input and output are reversed. In this appendix ten different examples will be discussed of the numerous point-to-point relations in passive electrical, mechanical and acoustical systems and in interconnected combinations of them. Also a heuristic procedure is outlined, which is helpful in deriving such relations. More extensive discussions on the applications can be found in the list of references added to this appendix.

A.1 Examples

In Fig. A.1 ten examples of point-to-point reciprocity relations are given, which are briefly discussed below.

System 1: This is a mechanical system. Interchanging of input and output position may be profitable, for example, when position 1 is not accessible for an appropriate exciter or when the background noise at position 1 is much lower than at position 2. Notice that the conjugate pairs of quantities F_1, v_1' and F_2, v_2' represent the same vibrational degree of freedom in each case ('conjugate' means that their product has the dimension of power).

System 2: This system has the excitation and response positions on mechanical structures, but (part of) the transmission occurs through an acoustical subsystem. The same reciprocity relations are valid as for system 1, but here another variant is shown. The original excitation is by a torque M_1 and the response of interest is the velocity v_2 in a direction normal to the structure. When source and receiver position are interchanged, the excitation and response quantities are to be altered as well. In addition to the possible advantages mentioned under system 1, the advantages for this example are that the original torque excitation is more complex than the force excitation in the reciprocal experiment, whereas the angular velocity φ_1' at position 1 is relatively simply obtained from measuring the difference of two closely spaced translational velocities.

System 3: This is an acoustical system with omnidirectional point sources and receivers. The possible advantages of reciprocal experiments are the same as those mentioned under system 1.

System 4: This reciprocity relation is the same as for system 3, but now the original source consists of an infinitesimal part of a vibrating structure (see discussion on source strength characterization with correlated monopoles [1]).

System 5: This reciprocity relation is for a dipole source and may be derived from a linear combination of two closely spaced systems like system 3. This example shows that the well-known relation of system 3 may be generalized to sources and receivers other than monopoles. The requirements are that the receiver in the reciprocity experiment has the same directivity and orientation as the original source and vice versa. The advantage is often, that a wide-band (i. e. acoustically compact) dipole or multi-pole receiver is easier to compose than the corresponding source. The torque examples in systems 2 and 7 form the mechanical analogue of the acoustical dipole, because the torque is modelled as a force couple

System 6: This reciprocity relation facilitates the transfer function measurement for acoustical excitation of a structure (e. g. in sound insulation studies). The excitation field, which is the summation of the incident, reflected and diffracted fields, is discretized and phase relations are retained. By appropriate transducers, low-pass wavenumber filtering can be applied to reduce the amount of transfer function measurements needed [A.15-A.16].

System 7: This is one variant of a frequently applied relation, in which the often complex mechanical excitations are replaced by simpler acoustical excitation. Advantages are similar to those mentioned under system 1 and 2. Using, for example, a Laser Doppler Velocimeter, would make the reciprocal experiments completely contactless. This may be a major advantage on miniature structures.

System 8: This interchanging of input and output may be profitable, because a wide-band monopole receiver is easier to obtain than the corresponding source. The dual mode application of the reversible electrical-mechanical transducer may reduce the measurement errors when determining unknown acoustical source strengths Q_1 [A.17].

System 9: If transfer functions in both 'directions' are measured, this two-way experiment may be used to check whether or not the intended mechanical excitation with a pure point force is contaminated by additional unwanted components. This is to be avoided when measuring point mobilities or power injection by point forces.

System 10: Both at the input and at the output terminal a reversible electromechanical or electro-acoustical transducer is included. Two-way experiments may be used to check whether or not the intermediate mechanical-acoustical system is reciprocal, within the amplitude range of excitations and responses covered by both experiments.

A.2 Heuristic method for deriving reciprocity relations

Electrical systems: The simplest representation of a network composed of linear passive elements (resistors, inductances, capacitors, transformers) is a two-port system. Let e_1, i_1 and e_2, i_2 be the conjugate variable pairs for the voltages and currents at the input port no. 1 and output port no. 2. Harmonic signals are assumed and the symbols represent complex phasors. Consider two different sets of voltages and currents (e_1, i_1, e_2, i_2) and (e_1', i_1', e_2', i_2'), resulting respectively from successive excitation at port no. 1 and at port no. 2. Then the reciprocity theorem for electrical networks says that:

$$e_1' i_1 + e_2' i_2 = e_1 i_1' + e_2 i_2'. \qquad \text{(A.1)}$$

From Eq. (A.1) three special reciprocity relations as in Eqs. (A.2a, 2b, 2c) follow. These correspond to the different 'boundary conditions', which are to be met in the equivalent 'direct' and 'reciprocal' experiments:

$$\left.\frac{e_2}{i_1}\right|_{i_2=0} = \left.\frac{e_1'}{i_2'}\right|_{i_1'=0}; \quad \left.\frac{i_2}{e_1}\right|_{e_2=0} = \left.\frac{i_1'}{e_2'}\right|_{e_1'=0};$$

$$\left.\frac{e_2}{e_1}\right|_{i_2=0} = -\left.\frac{i_1'}{i_2'}\right|_{e_1'=0}. \qquad \text{(A.2a, 2b, 2c)}$$

If the circuit has n ports instead of two, the theorem of Eq. (A.1) is replaced by:

$$\sum_{j=1}^{n} e_j' i_j = \sum_{j=1}^{n} e_j i_j'. \qquad \text{(A.3)}$$

Mechanical systems: The reciprocity theorems and reciprocity relations for mechanical systems are quite similar to those of electrical systems. The voltages and currents are replaced by forces and velocities. However, it may be necessary to characterize the 'state' at a single physical point by six vibrational degrees of freedom. For the characterization of a system between two physical points, then a 12-port model is appropriate, with the following pairs of conjugate orthogonal 'forces' and 'velocities' $(F_{1x}, v_{1x}; F_{1y}, v_{1y}; F_{1z}, v_{1z}; M_{1x}, \varphi_{1x}; M_{1y}, \varphi_{1y}; M_{1z}, \varphi_{1z}; F_{2x}, v_{2x}; \ldots; M_{2z}, \varphi_{2z})$. Using F_j and v_j as shorthand notation, both for translational and rotational components, the general reciprocity theorem for a 12-port can be written, in analogy with Eq. (A.3), as

$$\sum_{j=1}^{12} F_j' v_j = \sum_{j=1}^{12} F_j v_j'. \qquad \text{(A.4)}$$

Special relations can be derived in the same way as for the electrical system. Examples are:

$$\left.\frac{v_{2x}}{F_{1z}}\right|_{\substack{\text{other 11}\\ \text{'forces' zero}}} = \left.\frac{v_{1z}'}{F_{2x}'}\right|_{\substack{\text{other 11}\\ \text{'forces' zero}}};$$

$$\left.\frac{F_{2x}}{v_{1y}}\right|_{\substack{\text{other 11}\\ \text{'velocities' zero}}} = \left.\frac{F_{1y}'}{v_{2x}'}\right|_{\substack{\text{other 11}\\ \text{'velocities' zero}}}. \qquad \text{(A.5a, 5b)}$$

Because harmonic signals are supposed, all relations remain valid when velocities are replaced by accelerations.

Acoustical systems: In acoustical systems the reciprocity theorems and relations are again similar to the electrical ones. Now the voltages and currents are replaced by point pressures and point monopole volume velocities. The general theorem for an *n*-port is:

$$\sum_{j=1}^{n} p_j' Q_j = \sum_{j=1}^{n} p_j Q_j' \, . \qquad \text{(A.6)}$$

For a 2-port system, the special reciprocity relations are:

$$\left.\frac{p_2}{Q_1}\right|_{Q_2=0} = \left.\frac{p_1'}{Q_2'}\right|_{Q_1'=0}; \quad \left.\frac{Q_2}{p_1}\right|_{p_2=0} = \left.\frac{Q_1'}{p_2'}\right|_{p_1'=0};$$

$$\left.\frac{p_2}{p_1}\right|_{Q_2=0} = -\left.\frac{Q_1'}{Q_2'}\right|_{p_1'=0}. \qquad \text{(A.7a, 7b, 7c)}$$

***Interconnected systems*:** Reciprocity is valid in heterogeneous interconnected systems as well. The systems that will be briefly considered here, are combinations of mechanical structures and acoustical subsystems, but they may also include reversible electromechanical or electroacoustical transducers for experimental convenience.

Homogeneous n-ports. If all ports are of the same nature, i. e. either electrical or mechanical or acoustical, the same theorems and relations are valid as in Eqs. (A.1-A.7). Only sign reversal may occur if at one of the terminations an 'anti-reciprocal' reversible transducer is included as part of the system. Moving coil transducers are anti-reciprocal because they use the same permanent magnetic field in the 'source mode' and in the 'receiver mode' (see systems 8 and 9 in Fig. A.1).

Heterogeous n-ports. For mechanical-acoustical systems (see system 7 in Fig. A.1) the general reciprocity theorem for a *n*-port with *m* structural ports and *n-m* acoustical ports is in analogy with Eq. (A.3):

$$\sum_{i=1}^{m} F_i' v_i + \sum_{j=m+1}^{n} p_j' Q_j = \sum_{i=1}^{m} F_i v_i' + \sum_{j=m+1}^{n} p_j Q_j'. \qquad \text{(A.8)}$$

Of particular practical interest is the case with six structural ports at one point (i. e. six DOF) and with one acoustical port at the other point. Examples of reciprocity relations analogous to Eq. (A.2c) are:

$$\left.\frac{p_2}{F_{1x}}\right|_{Q_2=0} = -\left.\frac{v_{1x}'}{Q_2'}\right|_{\substack{\text{all 6 output}\\ \text{'forces' zero}}} ;$$

$$\left.\frac{p_2}{M_{1y}}\right|_{Q_2=0} = -\left.\frac{\varphi_{1y}'}{Q_2'}\right|_{\substack{\text{all 6 output}\\ \text{'forces' zero}}} . \qquad \text{(A.9)}$$

For electrical-mechanical systems between a structural point and an anti-reciprocal reversible transducer and for electrical-acoustical systems between an acoustical point and an anti-reciprocal reversible transducer (see Fig. A.1, systems 9 and 8), the reciprocity theorems are respectively as follows:

$$e_1' i_1 - \sum_{j=1}^{6} F_{2j}' v_{2j} = e_1 i_1' - \sum_{j=1}^{6} F_{2j} v_{2j}'; \qquad \text{(A.10)}$$

$$e_1' i_1 - p_2' Q_2 = e_1 i_1' - p_2 Q_2'. \qquad \text{(A.11)}$$

Special (anti-)reciprocity relations analogous to Eq.(A.2c) and Eq. (A.2a) respectively are:

$$\left.\frac{v_{2i}}{i_1}\right|_{\substack{\text{all 6 output}\\ \text{'forces' zero}}} = \left.\frac{e_1'}{F_{2i}'}\right|_{\substack{i_1'=0\\ \text{5 other input}\\ \text{'forces' zero}}} ; \qquad \text{(A.12)}$$

$$\left.\frac{p_2}{i_1}\right|_{Q_2=0} = -\left.\frac{e_1'}{Q_2'}\right|_{i_1'=0}. \qquad \text{(A.13)}$$

Remarks on 'boundary conditions': All point-to-point relations that have been derived in the above treatment, include requirements for the boundary conditions. For example, in Eq. (A.5a) the boundary conditions require that on the input side only one force is non-zero. On the output side the external loads, e.g. due to transducer inertia, are negligibly small, so that effectively free velocities are measured. Boundary conditions as implied in Eq. (A.5b) denote fixed degrees of freedom and are not normally practical. However, exceptions are, for example, resilient mounts[A.18] and lightweight structures which can easily be blocked with externally applied inertial loading.

A.3. References to appendix

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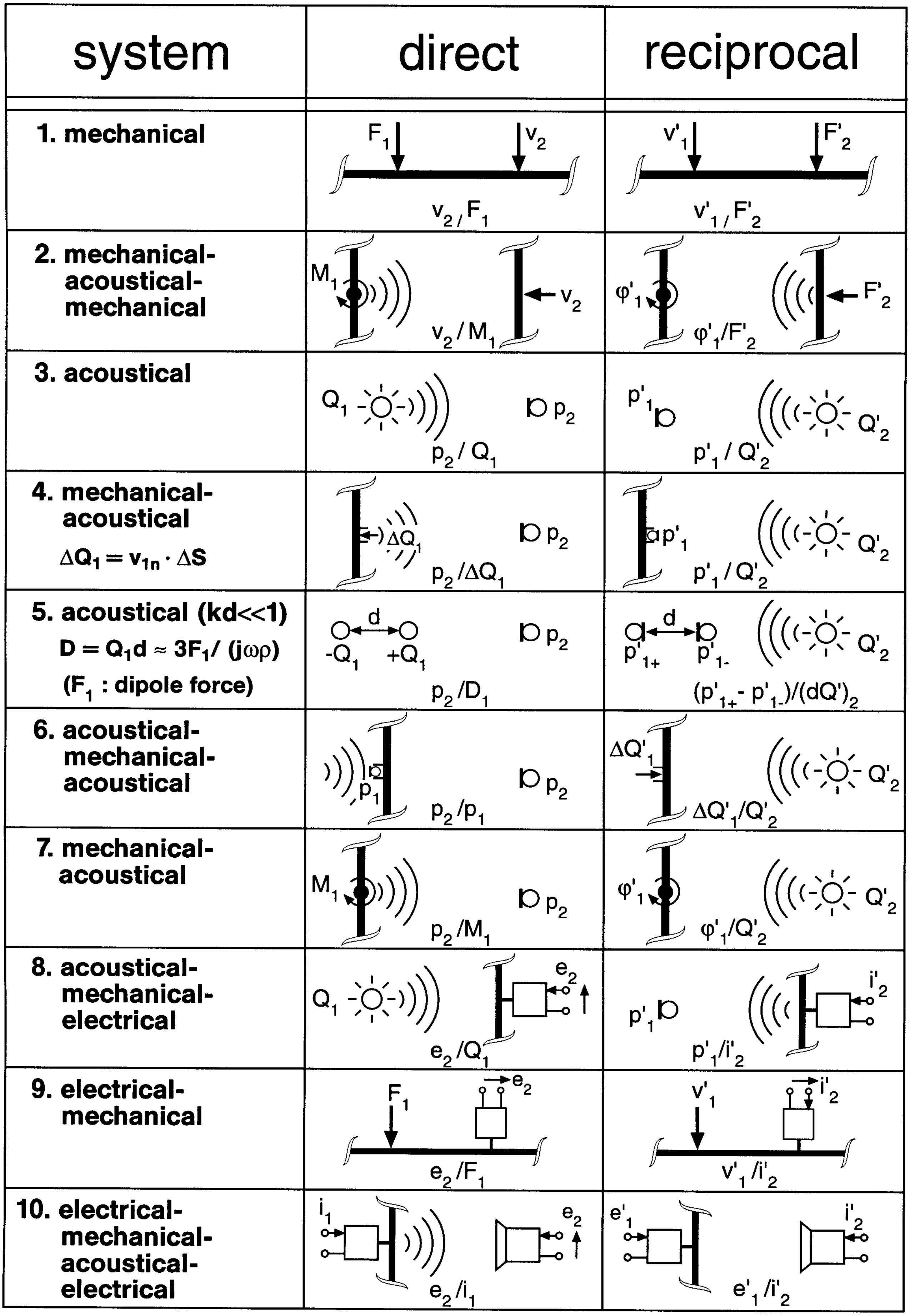

Figure A.1. Examples of equal point-to point frequency response functions in reciprocal systems.

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