Nonlinear Transverse Vibrations of a Slightly Curved Beam resting on Multiple Springs

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In this study, nonlinear vibrations of a slightly curved beam of arbitrary rise functions is handled in case it rests on multiple springs. The beam is simply supported on both ends and is restricted in longitudinal directions using the supports. Thus, the equations of motion have nonlinearities due to elongations during vibrations. The method of multiple scales (MMS), a perturbation technique, is used to solve the integro-differential equation analytically. Primary and 3 to 1 internal resonance cases are taken into account during steady-state vibrations. Assuming the rise functions are sinusoidal in numerical analysis, the natural frequencies are calculated exactly for different spring numbers, spring coefficients, and spring locations. Frequency-amplitude graphs and frequency-response graphs are plotted by using amplitude-phase modulation equations.

1. INTRODUCTION

Some beam elements of bridges, rails, and automotive industries are designed for the purpose of preventing impact by modern engineers. One of these elements is a curved beam model resting on an elastic foundation. Matter considered in these models, which have nonlinear behavior, is the resonance case of the system. If the system comes into a resonance state, the amplitudes increase dangerously, which is an unwanted case. Nonlinear problems of the model must be examined in order to prevent these cases that may occur at any time during vibration. For this reason, the linear part of the system must first be solved analytically and then the effects of nonlinearity should be added to the solutions. Thus, nonlinear vibrations of the system can be investigated. Before introducing the background of curved or pre-buckled beams, some studies related to our investigation must be mentined. Nayfeh and Mook reviewed and presented relevant works to the field up to 1979 in their book.[1] Cha derived governing equations for a linear elastica carrying a number of lumped masses, springs, and viscous dampers.[2] Albarracn et al. studied free vibrations of a uniform beam with intermediate constraints and ends that were elastically restrained against rotation and translation.[3] Wang and Qiao derived a general solution of the modal displacement of a beam with arbitrary discontinuities.[4] Wiedemann studied an arbitrary system of Euler-Bernoulli beams that were interconnected by arbitrary joints and subject to arbitrary boundary conditions.[5] Huang and Chen studied structures with multiple attachments that were subjected to axial forces and oscillations.[6] Regarding some assumptions in their model, they examined the remaining model with the pure buckling problem, the free vibration problem, and the general eigenvalue problem. Kelly and Srinivas investigated elastically connected axially-loaded beams, which may be attached to a Winkler foundation.[7] Wang et al. studied the nonlinear interaction of an inextensional beam on an elastic foundation with a three-to-one internal resonance.[8]

In some studies, the beam was assumed to have a rising function so the curvature effect on vibrations of the beam could be investigated. Some of these studies were such that Rehfield derived the equations of motion of a shallow arch with an arbitrary rise function and studied the free vibrations approximately.[9] Singh and Ali studied a moderately thick clamped beam with a sinusoidal rise function by adding the effects of transverse shears and rotary inertia.[10] Hajianmaleki and Qatu focused on the last two decades of research (1989-2012) done on vibration analysis.[11] They reviewed various beam theories such as thin (or classical), thick (or shear deformation), layerwise beam theories, and different methods for solving equations of motion, such as the transfer matrix method and the finite element method. Tien et al. studied the dynamics of a shallow arch subjected to harmonic excitation.[12] In the presence of both external and 1:1 internal resonance, he examined the bifurcation behavior of the shallow arch system. Using two beam elements, one has three degree-of-freedom and other four. Krishnan and Suresh studied the static and free vibrations of curved beams.[13] Oz et al. examined a simply supported and slightly curved beam resting on an elastic foundation with cubic non-linearities.[14] Considering free-undamped and forced-damped vibrations, they analyzed the effects of the elastic foundation, axial stretching, and curvature on the vibrations of the beams. Using a systematic theoretical procedure, Lin presented a static analysis of extensional circular-curved Timoshenko beams with general nonhomogeneous elastic boundary conditions and found the generalized Green function of differential equations.[15] For a general state of non-uniform initial stress, Chen and Shen derived the virtual work expressions of initially stressed curved beams.[16] They investigated the influence of arc segment angles, elastic foundations, and initial stresses on natural frequencies. Nayfeh et