Exact Solution for a Free Vibration of Thermoelastic Hollow Cylinder Under GNIII Model

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The exact analytic solutions are obtained with the use of the eigenvalue approach for a free vibration problem of a thermoelastic hollow cylinder in the context of Green and Naghdi theory (GNIII). The dispersion relations for the existence of various types of possible modes of vibrations in the considered hollow cylinder are derived in a compact form and the validation of the roots for the dispersion relation is presented. To illustrate the analytic results, the numerical solution of various relations and equations has been carried out to compute the frequency, thermoelastic damping and frequency shift of vibrations in a hollow cylinder of copper material with MATHEMATICA and MATLAB software.

1. INTRODUCTION

In the literature concerning thermal effects in continuum mechanics, several parabolic and hyperbolic theories for describing the heat conduction were developed. These hyperbolic theories were also called theories of second sound and there the flow of heat was modelled with finite propagation speed, which contrasts with the classical model based on the Fouriers law leading to infinite propagation speed of heat signals as in.[1–9] Green and Naghdi[10,11] proposed GNII and GNIII, which is a generalized thermoelasticity theory based on entropy equality rather than the usual entropy inequality. An important feature of this theory, which was not present in other thermoelasticity theories, was that it does not accommodate the dissipation of thermal energy. GN theory seems to be idealistic from a physical point of view. The genesis lies in the fact that the thermoelastic model of the GN theory was an idealized material model. During the last years, different problems were considered by using Green and Naghdi theories, as in Abd El-Latief et al.,[12] Youssef,[13] Mukhopadhyay et al.,[14] Sharma et al.,[15] Prasad et al.,[16] Othman et al.,[17] Abbas,[18] and Abbas et al.[19] A survey article of representative theories in the range of generalized thermoelasticity is given by Hetnarski and Ignaczak.[20]

The vibrations in thermoelastic materials have many applications in various fields of science and technology, namely aerospace, atomic physics, thermal power plants, and chemical pipes. The cylinders were frequently used as structural components and their vibrations were obviously important for practical design. Abbas studied the natural frequencies of a poroelastic hollow cylinder.[21] Abd-alla and Abbas investigated the magnetoelastic longitudinal wave propagation in a transversely isotropic circular cylinder.[22] Mykityuk studied the thermoelastic vibrations of a thick-walled cylinder of time-varying thickness.[23] Zhitnyaya analyzed an uncoupled problem of the thermoelastic vibrations of a cylinder.[24] Marin and Lupu studied the harmonic vibrations in thermoelasticity of micropolar bodies.[25] Erbay et al. investigated thermally induced vibrations in a generalized thermoelastic solid with a cavity.[26] Sharma et al. solved the vibration analysis of a transversely isotropic hollow cylinder by using the matrix Frobenius method.[27] Nayfeh and Younis presented a model for thermoelastic damping in microplates.[28,29] Rezazadeh et al. studied the thermoelastic damping in a micro-beam resonator using modified couple stress theory.[30]

The present article is devoted to study the frequency, frequency shifts and damping due to thermal variations in homogenous isotropic hollow cylinder, in the context of Green and Naghdi of type III model of non-classical (generalized) thermoelasticity.

2. BASIC EQUATION AND FORMULATION OF THE PROBLEM

Following Green and Naghdi, the basic equations of the thermoelasticity theory for homogeneous isotropic material in the absence of body forces and heat sources were considered as the equations of motion[10,11]:

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}; \tag{1}$$

where ρ was the density of the medium, t was the time, σ_{ij} were the components of stress tensor, and u_i were the components of displacement vector. The equation of heat conduction is:

$$\left(K^*_{ij} T_{,j} + K_{ij} \dot{T}_{,j}\right) = \frac{\partial^2}{\partial t^2}\left(\rho c_e + \gamma T_0 e\right); \tag{2}$$

where T is the temperature, c_e was the specific heat at constant strain, K_{ij} was the thermal conductivity, K^*_{ij} was the material constant characteristic of the theory, T_0 was the reference temperature; $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t was the coefficient of linear thermal expansion. The constitutive equations were given by:

$$\sigma_{ij} = 2\mu e_{ij} + \left[\lambda e - \gamma(T - T_0)\right]\delta_{ij}; \tag{3}$$

with $e = e_{ii},\ i,j = r,\theta,z$, where λ, μ were the Lame's constants and δ_{ij} was the Kronecker symbol.