
Vibration Analysis of Cracked Beams Using Adomian Decomposition Method and Non-Baseline Damage Detection via High-Pass Filters

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The Adomian decomposition method (ADM) and high-pass filters are employed in this study to investigate the free vibrations and damage detection of cracked Euler-Bernoulli beams. Based on the ADM and employing some simple mathematical operations, the closed-form series solution of the mode shapes can be determined for beams consisting of an arbitrary number of cracks under general boundary conditions in a recursive way. Then, a high-pass filter is used to extract the irregularity profile from the corresponding mode shape. The location and size of the cracks in the beam can be determined by the peak value of the irregularity profile. The numerical results for different locations and depths of cracks on the damaged beam under different boundary conditions are presented. The results show that the proposed method is effective and accurate. The experimental work for aluminium cantilever beams with one and two cracks was performed to verify the proposed method. The successful detection of cracks in the beam demonstrates that the proposed method has great potential in crack detection of beam-type structures, as it is simple and does not require the mode shapes of an uncracked beam as a baseline.

1. INTRODUCTION

Recently, many vibration-based damage detection techniques have been developed due to their non-destructive nature.¹⁻³ The popularity of these techniques is based on the fact that the loss of stiffness due to structural damage changes the dynamic response of the structure. With these techniques, damages can be detected by monitoring the vibration parameters, such as damping ratios, natural frequencies, and mode shapes.

Mode shapes and/or their derivatives are generally used to predict the location and the size of the damage rather than natural frequencies. Because the natural frequencies are the global features of the structure, it is difficult to determine the damage location with a frequency-based method.¹ Since the 1990s, a lot of damage detection algorithms based on mode shape have been proposed for damage detection and localization.^{1,2,4,5} Most of these methods require knowing the mode shapes of the health structures, which are difficult to obtain (and sometimes impossible), in order to establish a baseline for damage detection.

If the applicability of the mode shaped-based damage detection approach could be extended by eliminating the need for the baseline mode shapes, this approach would be significantly expanded in structural damage detection applications. Because of this potential, the non-baseline mode shape-based damage detection approaches have received more and more attention. Recently, Qiao and Cao⁶ calculated the fractal dimension (FD) and waveform fractal dimension (WFD) of the mode shape from a cracked beam to determine the damage location

and quantification. Ismail, et al.⁷ used fourth derivatives of the mode shapes to directly identify the location of damage for reinforced concrete beams. The application of 1-D and 2-D wavelet transform methods to displacement mode shape for damage detection of beam and plate structures have also been extensively investigated.^{8,9}

Ratcliffe, et al.^{10,11} proposed the gapped smoothing method (GSM) and the global fitting method (GFM) for damage detection. The GSM and GFM do not require data from the undamaged structure. By applying GSM or GFM to the mode shapes of the damaged structures, a smoothing curve, which could be regarded as a substitution for the mode shape from the undamaged structure, can be extracted. The GSM and GFM later used the operating deflection shape and its curvature data, and were extended to directly use two-dimensional COS data for damage detections.¹²⁻¹⁴

Recently, Wang and Qiao¹⁵ proposed an irregularity-based method to detect the cracks in beam structures. In this method, The Gaussian filter and triangular filter are applied on the mode shapes to extract the irregularities from the mode shape of the cracked beam, indicating the damage in the structure. The irregularity-based method was extended to detect the delamination in composite laminated beams and plates.^{16,17}

In this study, high-pass filters are used to extract the irregularities from the mode shapes and determine the damage situation in a beam. The aim of the paper presented here is twofold. Firstly, mode shapes for a beam with an arbitrary number of cracks under general boundary conditions are determined by the Adomian decomposition method (ADM).¹⁸⁻²² Using the