Developing Vibration Equations of an Orthotropic Wrapped Shell, Considering Residual Stress Effects; A Mathematical Approach

Amirali Sadeqi

Department of Mechanical Engineering, Shahid Chamran University of Ahvaz, 1714, Ahvaz, Iran.

Mojtaba Mahzoon and Mohammad Hassan Kadivar

Department of Mechanical Engineering, Shiraz University, Shiraz, Iran.

(Received 20 November 2013; accepted 8 October 2015)

In this paper, vibration equations of an orthotropic, thin rectangular plate wrapped around a porous drum are developed, considering residual stress effects. It is assumed that the plate is subjected to tension from both opposite sides and wrapped continuously around a cylindrical drum so that the wrapped portion behaves like a circular cylindrical shell. First of all, the Lame' parameters, required to constitute the geometry relations, are established for typical cylindrical shallow shell in cylindrical coordinate system. Then, the equations of motion are derived by utilizing the stored strain energy principle based on the Love assumptions. Finally, a set of more complete vibration equations is introduced by applying the simplifications of the Donnell-Mushtari-Vlasov theory. The equations derived under more stringent and precise assumptions are compared with those obtained and available in literature, and the discripancies are highlighted. The present study only aims to mathematically develop the governing relationships, where a numerical solution separately done by the authors can be found in other literature in which vibrational behavior has been completely discussed for moving and stable anisotropic wrapped plates.

NOMENCLATURE

A_1, A_2	gyroscopic inertia matrix
C_{ij}	Stiffness matrix corresponding to extension
D_{ij}	Stiffness matrix corresponding to bending
e_1, e_2, e_3	Unit vectors
g^1, g^2	Decomposed variables
h	Thickness of plate
K_1 to K_4	Symmetric stiffness matrix
k	Curvature
L_1, L_3	Length of flat segments
L_2	Length of wrapped or curved segment
M	Resultant moment, mass matrix
m	Number of terms in approximation function, longitudinal
n	Number of terms in approximatio function, lateral
P_0	Air pressure
P, P'	Typical vector points
P_{ij}	Dummy coefficients or stiffness ratios
Q, $\overline{Q}$	Stiffness matrices principal & material direction
$Q(t)$, $\dot{Q}(t)$	Reduced order matrix of spatial vector q
$Q_{\theta z}$, Q_{yz}	Transverse shear forces
q	Time dependent variable(vector)
q^r	Static load
$R(x,y)$	Radius of the drum or wrapped region
R	Redial direction, Residual index
$\overline{r}$	Displacement vector
T	Tension
U	Potential energy
u	Displacement in x direction
v	Displacement in y direction
W	Spatial or time independent variable (matrix)
w	Transverse displacement in z direction
X, Y, Z	Cartesian directions
α_1, α_2, α_3	Orthogonal curvilinear coordinates
ε	Strain
$\Phi(\theta, y)$	Spatial or time independent variable of Airy Function
ϕ	Airy Function
φ	Direct angle with X direction
θ	Tangential direction
θ_w	Wrapping angle
ρ	Mass density
σ^r	Residual stress
υ	Poisson's ration
ω	Excitation or response frequency
Ψ^1_{ij}, Ψ^2_{ij}	Approximation functions
ξ	Local coordinate in longitudinal direction
$\nabla^2_k(\cdot)$, $\nabla^2_r(\cdot)$	Second order Laplace operators

1. INTRODUCTION

Wrapped plates are widely used in many industries, such as manufacturing of papers, foils, and magnetic films; conveyer belt systems and band saw blades.[1–12] The general schematic of the application is shown in Fig. 1. The safety of relatively thin products, such as newspapers and webs, as well as their manufacturing appliance, has drawn the attention of engineers in recent years to make a design mechanism that is more efficient and versatile.

Vibration analysis of plates and shells during translation has been turned to an essential process in order to extract the modal properties and to prevent possible damages or failure. As shown in Fig. 1, because of the shape of the drum circumference, the mid part of the plate behaves like a cylindrical shell. It is also supposed that the pressurized air exits from the