On the Instabilities in a Switchable Stiffness System for Vibration Control

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A strategy for vibration control in which the stiffness is switched on and off between a minimum and a maximum value within each oscillation cycle is considered in this study. The strategy has been shown to help greatly in decreasing residual vibrations, thus increasing the effective damping ratio in lightly-damped systems. This work explores the effect of the delay during the stiffness switching. In this study, it is predicted theoretically how a certain value of delay could cause instabilities, and experimental results are presented.

1. INTRODUCTION

Mechanical vibration is an undesirable condition that could lead to fatigue, noise, damage, and other harmful effects to structures, machines, and humans. There are different methods for vibration suppression, either through control of the vibration source, structural modifications, or vibration isolation. The vibration isolation method involves the use of a resilient element located between the vibration source and the receiver, normally modelled mathematically as an elastic element and viscous damper in parallel.[1] When the properties of these elements, i.e. damping and stiffness, have a fixed value, the isolator is said to be passive. However, there are some isolators in which the properties are able to change in real time depending upon the excitation, the so-called semi-active vibration isolators. Lately, there has been a growing interest in the use of semi-active isolation systems mainly for random and deterministic vibration, while few works exist on shock isolation. Most of the work has been done in the field of variable damping; for instance, some switchable or semi-active damping strategies based on the skyhook damper concept have been studied,[2] and Waters, et al. showed that reducing the damping to a lower value during a shock input can lead to better isolation performance.[3]

In the field of switchable stiffness strategies, Winthrop presented an important review in which different methods to achieve variable stiffness were documented.[4] A control strategy for transient vibrations was proposed by Onoda[5] considering an on/off logic aimed to extract energy, based on the switchable stiffness concept presented by Chen for structural vibration control.[6] A resetting technique was considered by Jabbari, et al.[7] and Leavitt, et al.,[8] also based on switchable stiffness with the objective of extracting energy from a mechanical system while having a high stiffness value at all times. Switchable stiffness control has recently been investigated theoretically and experimentally by the authors of this paper, as a means of energy dissipation in lightly damped systems.[9,10] The strategy developed by Ledezma, et al.[9] comprises a mass supported by two springs, one of which can be disconnected. Switching in and out of the spring involves a two-stage control strategy — stiffness control during the shock to reduce the maximum response of the payload, and reduction of the residual vibration after the shock has occurred. The theoretical simulations presented demonstrate that it is possible to obtain better shock isolation by switching the stiffness in lightly damped systems, and this concept was demonstrated experimentally.

This paper explores the second part of the strategy presented previously by the authors,[9,10] namely the residual control of vibrations, which involves switching in real time the actual stiffness of the system. It was found that a delay in the switching could lead to instabilities. Time delays in active vibration control may result in the unstable motion of a controlled oscillating system and hence limit the development and application of vibration control. Haiyan,[11] and later Gu,[12] summarized the recent studies on the dynamics of controlled systems with time delay. Due to the possibility of instabilities as a result of delays, this work gives an insight into the stability of the system, prediction of the maximum delay, and some of the practical issues involved.

2. SWITCHABLE STIFFNESS BACKGROUND THEORY

The switchable stiffness strategy presented by the authors is described briefly here, focusing on the residual vibration control. It is important to mention that residual vibration is the

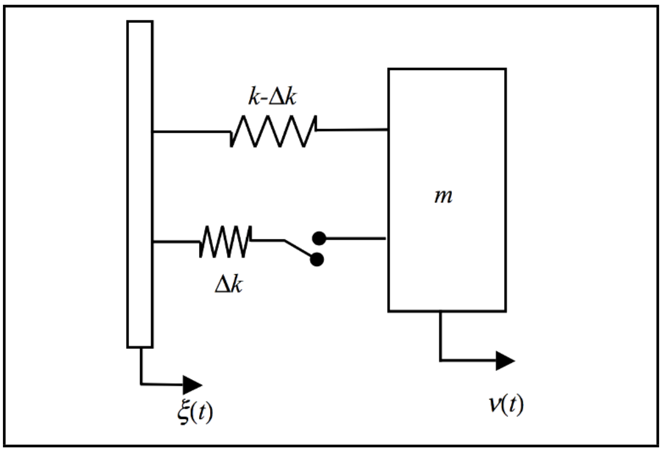

Figure 1. Schematic of a SDOF system with switchable stiffness under excitation $\xi(t)$ applied to the base. The two stiffnesses in parallel have stiffness Δk and $k - \Delta k$, respectively, and m is the mass with absolute displacement v.

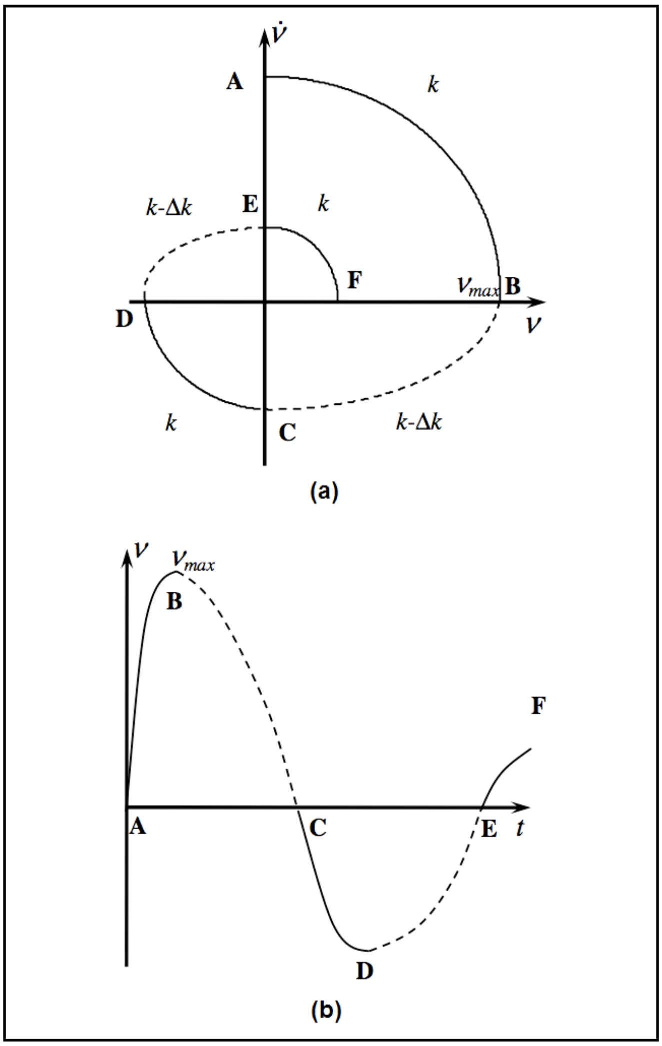

Figure 2. Effect of the switchable control strategy on the residual vibration. (a) Phase plane plot, and (b) Time history of the displacement response. (—— High stiffness; - - - - - Low stiffness).

result of a transient excitation, such as a pulse, i.e. a versed sine pulse. The analysis of shock isolation is normally divided in two stages: the forced response during the shock pulse, and the subsequent free vibrations at the natural frequency of the system. This paper focuses on what happens after the pulse, and it is assumed for simplicity that an impulse is applied when the system is at rest, so there is an initial velocity applied. The switching strategy considers an undamped system supported by two springs, one of which can be disconnected during free transient vibration, in order to quickly dissipate the energy stored by the elastic element without adding an external damping mechanism, as shown in Fig. 1. The control strategy is given by

$$k_{\text{effective}} = \begin{cases} k & \text{for } v\dot{v} \geq 0 \\ k - \Delta k & \text{for } v\dot{v} < 0 \end{cases}; \qquad (1)$$

where $\dot{v}$ is the velocity of the suspended mass. The stiffness is maximum and equal to k when the product $v\dot{v}$ is positive, and it is minimum and equal to $k - \Delta k$ when $v\dot{v}$ is negative. As a result, the secondary spring, Δk, is disconnected when the absolute value of the displacement of the mass is a maximum. It is connected again when the system passes through its equilibrium position. The effective stiffness change in the system is quantified with the stiffness ratio, defined as $\sigma = \frac{\Delta k}{k}$. This parameter varies between 0 and 1, where 0 means no stiffness reduction, and 1 means total stiffness removal. The schematic model is depicted in Fig. 1, where a base excitation $\xi(t)$ is applied and the response is $v(t)$. It is also important to introduce the mean period of the system, given by $T_{\text{m}} = \frac{T_{\text{on}}}{2} + \frac{T_{\text{off}}}{2}$, where T_{on} is the natural period for the ON state, i.e. high stiffness, and T_{off} is the period for the OFF state, i.e. low stiffness.

The typical behaviour of the switching strategy can be easily explained with the phase plane plot presented in Fig. 2(a) showing the switching and how the energy is dissipated at every stiffness reduction point. Figure 2(b) depicts a time history corresponding to this example. The comprehensive study of this strategy presented in a previous paper[9] concludes that a greater stiffness reduction leads to greater rate of decay of the residual vibrations.

3. EXPERIMENTAL DEVICE FOR VIBRATION CONTROL USING SWITCHABLE STIFFNESS

An experimental prototype was designed to validate the theoretical strategy discussed before.[10] Considering the results previously referenced, the experimental model has to be capable of achieving a high stiffness change — at least a factor of two — and perform this change rapidly, i.e. four times during a full oscillation cycle. It was also designed bearing in mind that the system should behave as a lightly-damped single degree-of-freedom system, for which predictions had been previously obtained.

To create a system with these characteristics, an electromagnetic element was used to achieve a switchable stiffness. The experimental system is shown in Fig. 3; a schematic of the system is given in Fig. 3(a), and a photograph of the system is shown in Fig. 3(b). The device comprises two disc-shaped neodymium magnets suspended between two electromagnets using four tensioned nylon wires attached to the main frame, providing additional stiffness in parallel with the stiffness given by the magnets. The total isolated mass (payload) in the experimental system was 0.0753 kg.

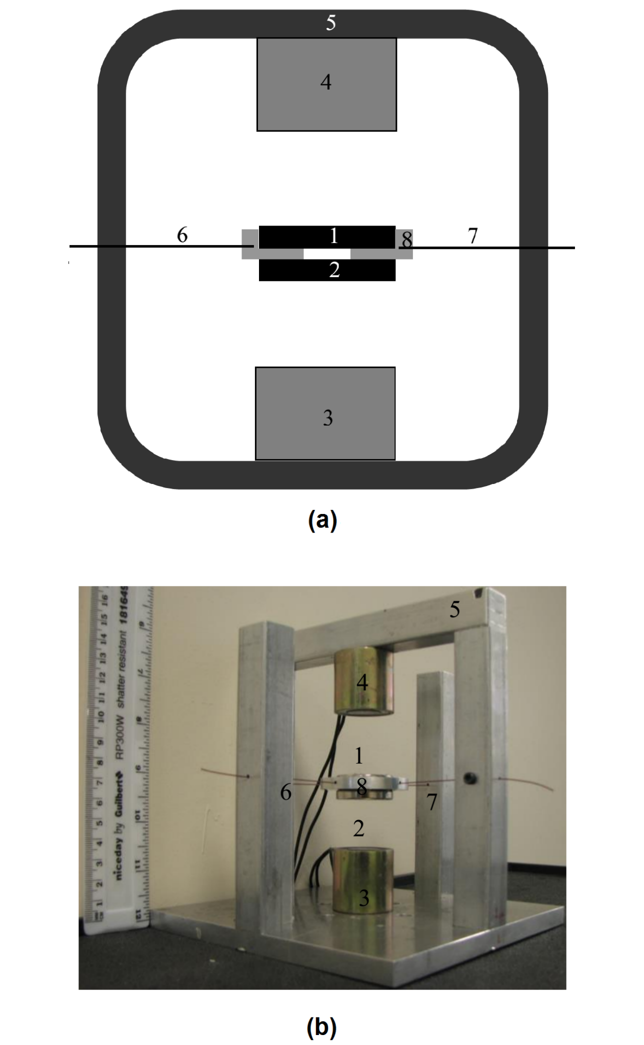

Figure 3. (a) Diagram of the switchable stiffness experimental system in the vertical position. The permanent magnets (1, 2) are suspended between two electromagnets (3, 4) using four wires (6, 7) that also join the magnet to the main frame (5). The permanent magnets are held by an aluminium disc (8). (b) Photograph of the rig.

The low stiffness state was achieved when the electromagnets are turned off, whilest the high stiffness state was obtained by applying a constant DC voltage of 12 V to the electromagnet, using a Hameg triple voltage source HM7042-5. The properties of the system were measured by attaching the device to an electrodynamic shaker with a random signal generated by a Data Physics Mobilizer analyser through a power amplifier. The response was acquired using two PCB teardrop miniature accelerometers type 352C22, and the frequency response function was measured. It was found that the maximum and minimum values of the natural frequencies were 17.75 Hz and 12.75 Hz, respectively, corresponding to an effective stiffness change of 48%. The full results for these properties and the procedure are available in a previously published paper.[10] It is also important to mention that the magnetic forces are nonlinear by nature, and as a result, the oscillation of the system was kept within a linear range for which coherence and static measurements were performed, as stated in previous work.[10]

For the control of residual vibrations, an electronic circuit was designed in order to perform the switching strategy given

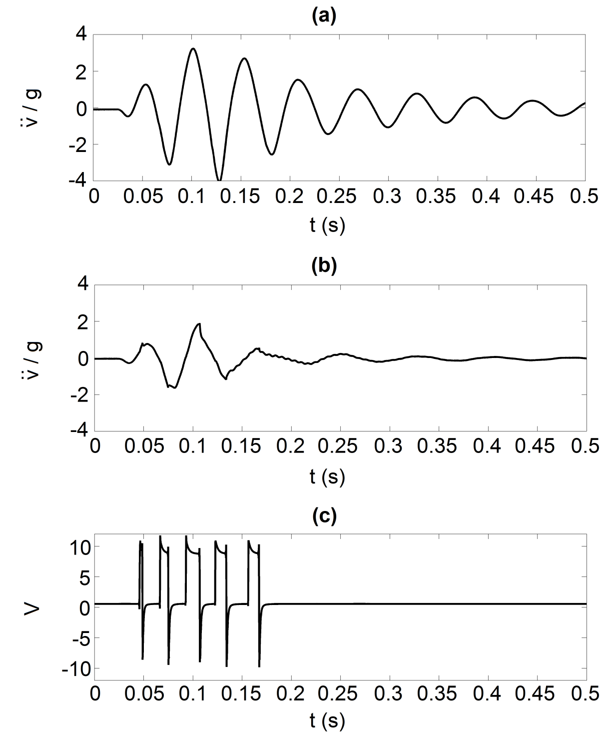

Figure 5. Switching stiffness response for residual vibration suppression. (a) Passive acceleration response for high stiffness state, (b) switching response and (c) the voltage supplied to the electromagnets. All acceleration time histories are normalized by g.

by Eq. (1). The acceleration signal measured in the suspended mass was amplified and then split into two in order to have acceleration and velocity signals. Then, both signals were multiplied, and the product was compared with respect to zero. If the product is greater than zero, the voltage is set to zero. Otherwise, a voltage is applied to the electromagnets. A block diagram is shown in Fig. 4(a), while a circuit diagram is shown in Fig. 4(b).

The system was subjected to a very short pulse and then the responses were measured. An example is presented in Fig. 5. Figure 5(a) depicts the free vibration acceleration response of the passive system in the high stiffness state, which is taken as a basis for comparison. The switching response is presented in Fig. 5(b), and the voltage applied to the electromagnets is given in Fig. 5(c).

It can be seen from the voltage plot that the stiffness is switched from a high state to a low state twice during each cycle of vibration. The voltage supply turned off when the response was at a maximum, and turned on when the response passed through the static equilibrium position. It can also be seen that there were only four cycles of stiffness change. This was because the circuit was set to switch only when the input signals were at a minimum voltage level of 0.7 V. However, even though the stiffness switched during the first two cycles of vibration, the vibration decayed at a much faster rate than the passive case, as can be seen by comparing Figs. 5(a) and 5(b).

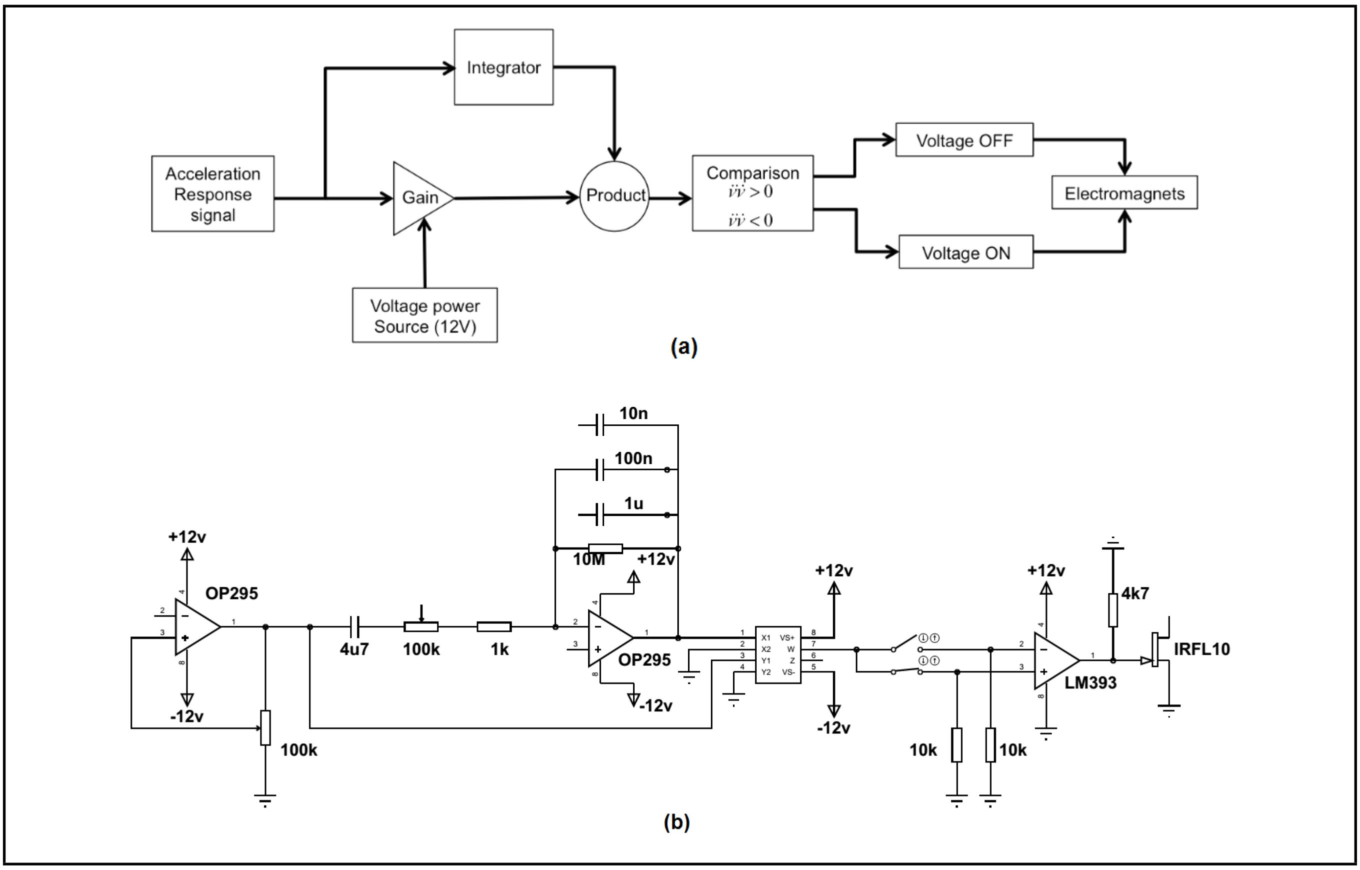

Figure 4. Electrical circuit used for the switching stiffness during residual vibration. (a) Block diagram. (b) Circuit diagram.

4. EFFECT OF DELAY ON THE CONTROL STRATEGY

4.1. Stability of the System

Using Lyapunov's direct method,[12] one can construct an energy-like function of the system and, examining its derivatives, the condition of stability can be evaluated. An energy function for the switching stiffness model can be written in terms of the total energy of the system, where the energy is considered after the disconnection of the secondary spring, being the sum of the kinetic energy of the mass and the potential energy stored in the effective low stiffness, as follows:

$$V = \frac{1}{2}m\dot{v}^2 + \frac{1}{2}(k - \Delta k)\,v^2. \tag{2}$$

The first time derivative of equation Eq. (2) can be written as:

$$\dot{V} = m\ddot{v}\dot{v} + k\dot{v}v - \Delta k\dot{v}v. \tag{3}$$

This can be further simplified using the equation of motion of the system given by $m\ddot{v} + k_{\text{effective}}v = 0$ and rewritten as

$$\dot{V} = -\Delta k\dot{v}v. \tag{4}$$

As stated in the control theory[13] as V is a positive definite function, and $\dot{V}$ is negative semidefinite, it can be said that the system is asymptotically stable. However, it is interesting to note the behaviour of the system when the control law is inverted as:

$$k_{\text{v}} = \begin{cases} k & \text{for } v\dot{v} \leq 0 \\ k - \Delta k & \text{for } v\dot{v} > 0 \end{cases}. \tag{5}$$

In this case, the energy function of the system is expressed as:

$$V = \frac{1}{2}m\dot{v}^2 + \frac{1}{2}(k + \Delta k)\,v^2; \tag{6}$$

and the derivative is given by:

$$\dot{V} = \Delta k\dot{v}v. \tag{7}$$

The derivative is a positive function, and as a result the system is unstable. This can also be observed in Eq. (6), as its second term means that energy is added to the system instead of being dissipated. In fact, at the stiffness reduction points, the system is going from a low energy state to a higher energy state. In a real system, this energy, which causes instabilities, must come from a certain source. This phenomenon will be discussed later.

4.2. Effect of Delay on Stability

An interesting point in the study for the stability of this particular strategy is to determine if a delay in the reduction or recovery of the stiffness will cause any possible instability condition. In order to investigate this, a numerical simulation was performed in MATLAB. The simulation comprises an iterative test that evaluates the amplitude of the system and compares it with the previous cycle. If the amplitude of the next cycle increases with respect to the previous one, the system will be unstable. The comparison is made for values of the stiffness ratio σ, i.e. the factor of stiffness reduction from 0 to 1, where 0 is no stiffness reduction and 1 means total stiffness removed. Then, a delay in the implementation of the strategy is

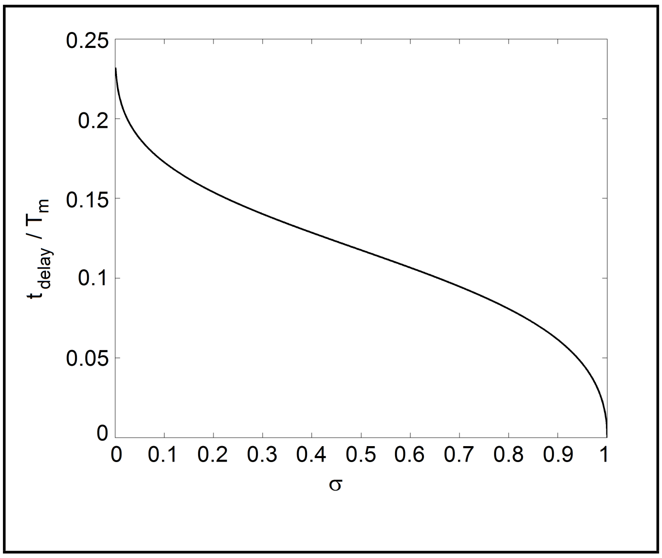

Figure 6. Maximum delay permissible in order to guarantee stability in the switching strategy presented as a function of the stiffness ratio σ. Values of delay above the curve will cause the system to become unstable.

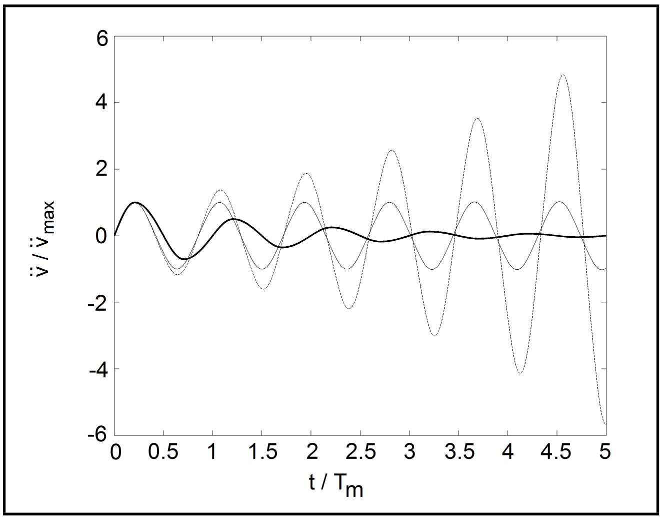

Figure 7. Theoretical acceleration response of the switching system showing the effect of delay in the switching stiffness strategy. This example shows no delay (continuous bold line), limiting value of delay $d = 0.1176\,T_{\rm m}$ (continuous thin line), and $d = 0.15\,T_{\rm m}$ (dotted line).

introduced on purpose. This delay is given as a fraction of the mean period of the system $T_{\rm m} = \frac{T_{\rm on}}{2} + \frac{T_{\rm off}}{2}$. The resulting plot is presented in Fig. 6.

Figure 6 represents the limiting value of delay that is permissible in order to achieve stability. The area below the curve represents a "safe" zone where the system is stable. However, it is important to note that although small delays will not cause instabilities, the performance of the system might decrease. The maximum value of delay depends upon the value of the stiffness reduction ratio σ. As the stiffness ratio increases, smaller delays are permitted. A time response example for $\sigma = 0.5$ is shown in Fig. 7, for three situations. The continuous bold line gives the response of the system with no delay, the continuous thin line represents the limiting value of delay $d = 0.1176\,T_{\rm m}$ for this value of the stiffness ratio, and the dotted line indicates a value exceeding the limit $d = 0.15\,T_{\rm m}$.

This phenomenon is attributed to two factors. First, as the stiffness reduction is not made at the point of maximum displacement, the energy dissipated is not maximized. Secondly, the stiffness recovery is performed after the point of equilibrium position. As a result, the secondary spring Δk has a certain deformation at the moment of reconnection. This means the secondary spring Δk has some energy stored that is returning to the system. If the delay causes the energy returned to be greater than the energy dissipated during the stiffness reduction, the amplitude will grow. Figure 8 shows the energy levels in the unstable system, showing that at some point the energy returned is higher than the energy removed. This will in fact violate energy conservation, which means that in a real system, the energy needed to cause the instability must come from an external source. In practice, this phenomenon has not been observed because the delay in the real-time implementation of the strategy is very small.

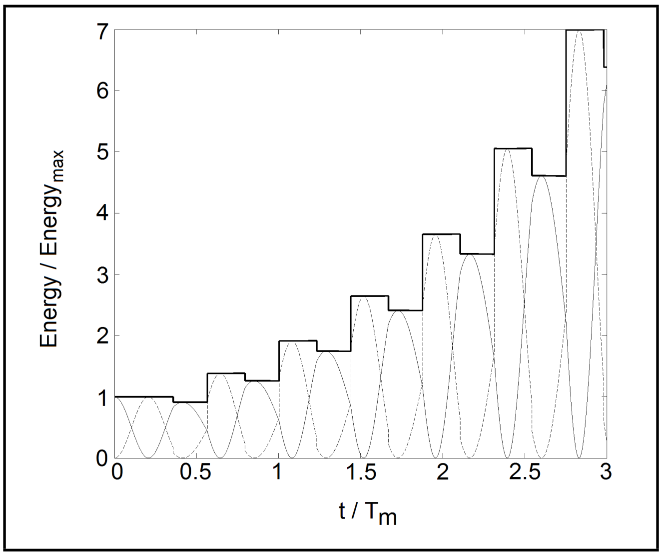

Figure 8. Energy levels in a switchable stiffness system with delay of $d = 0.15\,T_{\rm m}$ enough to cause instabilities. The bold line represents the total energy in the system, the thin line represents the kinetic energy, and the dotted line is for the potential energy.

If the delay is large enough to cause instabilities, in a real system the energy must come from an external source. For the experimental setup used in this project, the electromagnets as inductive devices are capable of storing and releasing energy resulting from sudden voltage or current changes. In practice, the presence of damping in the system could help to reduce the effect of delay and the possible instabilities.

Finally, an experimental test was performed considering the opposite control law as expressed by Eq. (5). Figure 9 shows the response of the actual system using the inverted control law. In this case, the amplitude of the system is bounded. However, the vibration of the system is sustained, and while the voltage is supplied, the amplitude never decays away. The energy responsible of this behaviour is believed to come from the electromagnets and the voltage source used in the system. As a result, a constant amplitude of vibration is sustained.

5. CONCLUSIONS

This study presents an overview of a switching stiffness strategy for vibration control. The strategy is based on a con-

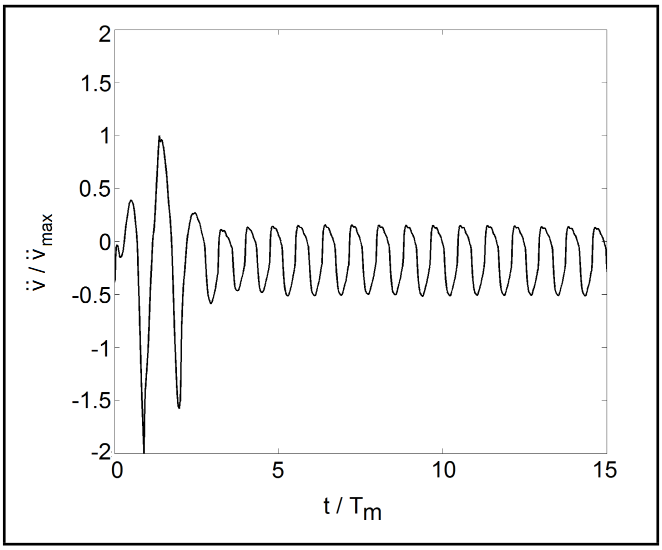

Figure 9. Actual response of the switching system considering the opposite control law as specified by Eq. (5). Time is normalised considering natural period of the system.

trol law that switches off the stiffness at points of maximum and minimum displacement and restores the stiffness when the isolated mass passes through equilibrium. Theoretical and experimental evidence was presented showing improved vibration control for lightly-damped systems.

The stability issues were discussed, and it was shown how a delay in the implementation of the strategy could lead to potential instabilities. The limiting value of permissible delay was calculated numerically and presented in the form of time histories showing the increase of amplitude due to energy cumulating. This phenomenon was not observed in the experimental rig, but it was shown experimentally how the inverted control law is indeed as unstable as the theory predicts. The development of an analytical study to predict the effect of delay is suggested for future work.

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