Review and Comparison of Variable Step-Size LMS Algorithms

Dariusz Bismor, Krzysztof Czyz and Zbigniew Ogonowski

Institute of Automatic Control, Silesian University of Technology, ul. Akademicka 16, 44-100 Gliwice, Poland

(Received 4 December 2013; accepted 11 February 2015)

The inherent feature of the Least Mean Squares (LMS) algorithm is the step size, and it requires careful adjustment. Small step size, required for small excess mean square error, results in slow convergence. Large step size, needed for fast adaptation, may result in loss of stability. Therefore, many modifications of the LMS algorithm, where the step size changes during the adaptation process depending on some particular characteristics, were and are still being developed.

The paper reviews seventeen of the best known variable step-size LMS (VS-LMS) algorithms to the degree of detail that allows to implement them. The performance of the algorithms is compared in three typical applications: parametric identification, line enhancement, and adaptive noise cancellation. The paper suggests also one general modification that can simplify the choice of the upper bound for the step size, which is a crucial parameter for many VS-LMS algorithms.

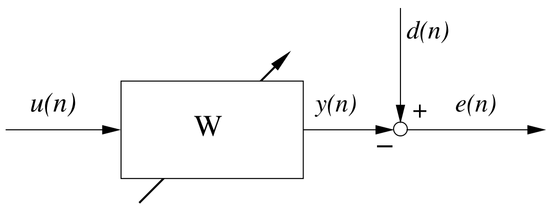

Figure 1. Adaptive filtering problem.

1. INTRODUCTION

In applications where adaptation is needed, the LMS algorithm is probably the most frequently used algorithm. It is simple, fast, and surprisingly robust. Despite its simplicity, the complete mathematical analysis of the LMS algorithm as well as exact rules for the step size adjustment are not currently known, which is probably due to its highly nonlinear character.[1] Therefore, new VS-LMS algorithms appear in the literature every few years with the aim to be useful in practical applications.

The basic block diagram illustrating the LMS algorithm operation is shown on Fig. 1.[1] The adaptive filter $\mathbf{W}$ is fed with the input sequence $u(n)$. The output of the filter, $y(n)$, is compared with the desired signal, $d(n)$, to produce the error signal, $e(n)$. The algorithm adjusts the filter to minimize the error.

If the adaptive filter is of finite impulse response (FIR) type, with the taps stored in a row vector:

$$\mathbf{w}(n) = [w_0(n)\ w_1(n)\ \ldots\ w_{L-1}(n)]^T ; \tag{1}$$

where T denotes transpose, the LMS algorithm updates the filter taps according to the well-known formula:[2]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{u}(n) e(n); \tag{2}$$

where μ is the step size parameter and $\mathbf{u}(n)$ is a row vector containing the input signal samples. The latter may be, depending on the application, of spatial type:

$$\mathbf{u}(n) = [u_0(n)\ u_1(n)\ \ldots u_{L-1}(n)]^T ; \tag{3}$$

or of temporal type, with regressive samples of the same input signal:

$$\mathbf{u}(n) = [u(n)\ u(n-1)\ u(n-2)\ \ldots u(n-L+1)]^T . \tag{4}$$

The problem with the step size choice can be summarized as follows. Large step size allows for fast adaptation, but also gives large excess mean square error (EMSE, see Section 4.1 for definition). Too large step size may lead to the loss of stability of the system using the LMS algorithm. On the other hand, too small step size gives slow convergence, and even if it results in small excess MSE, it cannot be accepted in many practical applications.

At this point a very important remark should be made about theoretical convergence of the LMS algorithm. First of all, there are different types of convergence,[3] e.g. convergence of the mean (the poorest), convergence in the mean, convergence in the mean square sense, etc. However, if convergence in the mean square sense of the LMS algorithm (2) is desired, and the algorithm operates in real conditions (not noise-free environment), such convergence can only be proved for the vanishing step size, i.e. for $\mu \xrightarrow{n\to\infty} 0$.[3,4] In other words, no constant step-size LMS algorithm can result in convergence in the mean square sense, or stronger. On the other hand, it is possible to bound the EMSE within certain limits, depending on the step size.

The idea of variable step-size is not new. Actually, the Normalized LMS (NLMS) algorithm may be considered as the first variable step-size modification of the LMS, and NLMS was proposed in 1967 independently by Nagumo *et al.*[5] and Albert *et al.*[6] Next VS-LMSes were proposed in 1986 by Harris *et al.*,[7] and by Mikhael *et al.*[8] Many VS-LMS algorithms were developed since then: the search for 'variable step LMS' in article titles only on Scopus or IEEEXplore returns more than 130 publications. The research in this field is by no means finished, new results are still being published.[9,10]