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# Vibrations of a Rotationally Restrained Circular Plate Resting on a Concentric Rigid Ring Support

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(Received 3 September 2013; accepted 3 October 2015)

In this paper, the vibrations of a circular plate with a rotationally restrained edge that has concentric rigid ring support are studied. The influences of the rotational restraint parameter and radius of internal rigid ring support on the vibration of the plate's natural frequencies are investigated. Frequencies for the first three modes of vibration are obtained and plotted graphically. The cross-over radius and the optimum location point of internal rigid ring support are determined. The results presented in this paper are from exact analysis, and hence can serve as standard values for estimating the accuracy of results obtained from various approximate methods.

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## 1. INTRODUCTION

In many branches of engineering—such as naval, automobile, and civil—continuous plates are extensively used. There exists a great deal of literature on the present subject area of circular plate vibrations, predominantly with free, clamped, and simply supported edges.<sup>1–4</sup> Leissa<sup>5–10</sup> has reported natural frequency results in many of his papers on continuous circular plates, and other researchers have reported results on the influence of internal rigid ring supports on the dynamic characteristics of circular plates.

Bodine<sup>11</sup> has premeditated the axisymmetric free vibrations of the circular plates, and Laura, et al.<sup>12</sup> presented useful results on the natural frequencies of axisymmetric modes of vibration. The case of the influence of rigid supports along with mode switching was studied by Bodine,<sup>13</sup> varying the values of the ring support radius and Poisson's ratio of the circular plate material. Ding Zhou<sup>14</sup> studied the free vibration of arbitrarily shaped plates with concentric ring elastic and/or rigid supports. In realistic circumstances, to fortify the load-carrying capability of the plate, occasionally internal stiffeners and/or concentric supports are used. In such cases, vibrational characteristics of the plate will vary significantly. Hence, the stiffeners' properties and concentric support should be included in the analysis of the circular plates. Many researchers studied the vibration characteristics of the circular plates with a range of boundary situation and internal intensification.<sup>15–26</sup> Wang<sup>26</sup> studied the problem of fundamental frequency of a circular plate on a ring and free boundary and presented the results for the fundamental frequency related to an anti-symmetric mode of vibration when the support radius is small. Many researchers studied the problem of vibrations of circular plates with concentric ring support, as well.<sup>27–30</sup>

Najafizadeh and Mirkhalaf Valashani<sup>31</sup> carried out the vibration analysis of circular plates that have an eccentric circular perforation and a free edge with an attached concentrated mass at any arbitrary position on the plate. The Rayleigh-Ritz variational method was applied to determine the fundamental natural frequency coefficient for the circular plates with the eccentric circular perforation and arbitrarily attached concen-

trated mass based on the classical plate theory (CPT). Mirkhalaf Valashani<sup>32</sup> utilized the Rayleigh-Ritz method to investigate the transverse vibration of clamped and simply supported circular plates with an eccentric circular perforation and attached concentrated mass. Wang<sup>33</sup> studied the vibration of a circular plate with an attached core and clamped, simply supported, free and sliding boundary conditions.

However, as we know, in practical industrial engineering situations, we seldom come across such ideal edge conditions. The review of research on the vibration of circular plates restrained against rotation can be found in the studies made by Laura, et al.,<sup>34</sup> Laura and Grossi,<sup>35</sup> Narita and Leissa,<sup>36</sup> Irie, et al.,<sup>37</sup> and Veera, et al.<sup>38</sup> It is well-established that the stipulation on an edge frequently tends to be in between the classical edge conditions (simply supported, free and clamped) and may be in contact with elastic restraints, such as rotational restraints.<sup>39–42</sup> However, there is no other research in the literature addressing the common boundary conditions with a rotational restrained edge at the plate's periphery.

In many practical situations such as bolted connections, the plate edge becomes something between a classical simply supported edge and a clamped edge. Often, the edge conditions can be simulated by using a rotational spring. This is exactly what is attempted in this paper. The main intention of this paper is therefore to study the effect of a rigid ring support radius along a concentric circle, and a plate with a rotationally restrained edge (shown in Fig. 1) using an exact method of solution approach. The natural frequencies of a circular plate for varying values of rotational restraint along the plate edge, and the ring support radius for a wide range of non-dimensional parameters, are presented in graphical form for use in design.

## 2. ANALYTICAL FORMULATION

Consider a plate of radius  $R$ , Poisson's ratio  $\nu$ , density  $\rho$ , modulus of elasticity  $E$ , and thickness  $h$ . Figure 1 shows a plate which has an outer boundary rotationally restrained and simply supported (radius  $R$ ), and a rigid ring support at radius  $bR$ .