Nonlinear Vibration Analysis of Flexible Hoisting Rope with Time-Varying Length

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The nonlinear vibration of a flexible hoisting rope with time-varying length and axial velocity is investigated. The flexible hoisting rope is modeled as a taut translating string with a rigid body attached at its low end. A systematic procedure for deriving the system model of a flexible hoisting rope with time-varying length and axial velocity is presented. The governing equations were developed by employing the extended Hamilton's principle considering coupling of axial movement and flexural deformation of the rope. The derived governing equations are nonlinear partial differential equations(PDEs) with time-varying coefficients. The Galerkin's method and the 4^{th} Runge-Kutta method were employed to numerically analyze the resulting equations. Further, the dynamic stability of the flexible hoisting rope was investigated according to the Lyapunov stability theory. The motions of an elevator hoisting system were presented to illustrate the proposed mathematical models. The results of simulation show that the dynamic motions of the flexible hoisting string are stable during downward movement but are unstable during upward movement. The proposed systematic procedures in analyzing the dynamic stability can facilitate further development in dynamic control of the flexible hoisting system in practice.

NOMENCLATURE

a	Axial acceleration of the string (m/s^2)
$\mathbf{A}, \mathbf{B}$	Matrix differential operators
$\mathbf{C}$	Damp matrixes
d	Diameter of the string (m)
E	Young's Modulus of the string (Pa)
E_k	Kinetic energy of flexible hoisting system (J)
E_e	Elastic strain energy of the string (J)
E_g	Gravitational potential energy of flexible hoisting system (J)
g	Gravitational constant (m/s^2)
$\mathbf{i}$	Unit vector along the x-axes
i	Integer
I	Inertia (m^4)
$\mathbf{I}$	$n \times n$ identity matrix
$\mathbf{j}$	Unit vector along the y-axes
j	Integer
k	Integer
$\mathbf{K}$	Stiffness matrixes
l	Length of the string (m)
m	Mass of rigid body (kg)
$\mathbf{M}$	Mass matrixes
n	Number of included modes
P	Longitudinal tension (N)
q_i	Generalized coordinates
$\mathbf{Q}$	Vectors of generalized coordinates
$\mathbf{R}$	Position vector of the string
$\mathbf{R_c}$	Position vector of the rigid body
t	Time (s)
T	Lyapunov candidate function
U	State vector
v	Axial velocity of the string (m/s)
$\mathbf{V}$	Velocity vector of the string
$\mathbf{V_c}$	Velocity vector of the rigid body
x	Spatial variable (m)
y	Transverse displacement of the string (m)
ζ	Transformed spatial variable
ε	Strain measure
ρ	Linear density of the string (kg/m)
λ_k	Eigenvalue ($k = 1, 2, 3, 4$)
ξ_k	Real parts of eigenvalue ($k = 1, 2, 3, 4$)
φ_i	Trial function used in Eq. (19)
δ_{ij}	Kronecker delta
ω_k	Imaginary parts of eigenvalue ($k = 1, 2, 3, 4$)
$\mathbf{\Lambda}$	Eigenvector

1. INTRODUCTION

Ropes with time-varying length are widely used in the hoisting industry such as mine hoists, elevators, cranes, etc; They are subject to vibration due to their high flexibility and relatively low internal damping characteristics.[1,2] Most often these systems are modeled as either an axially moving tensioned beam or as a string with time-varying length and a rigid body at its lower end.[3,4] It was reported that the vibration energy of the rope changed in general during elongation and shortening.[5–7] Zhang[8–11] and Bao[12,13] published a series of studies on vibration of a flexible hoisting system with arbitrarily varying length. Terumichi et al. assumed the velocity of the string was constant and studied the transverse vibrations of a string with time-varying length and a mass-spring system