Vibrational Power Flow Analysis of a Cylindrical Shell Using a Four-Point Technique

H. Salimi-Mofrad

Department of Mechanical Engineering, IOOC, Tehran, Iran.

S. Ziaei-Rad and M. Moradi

Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, Iran.

(Received 20 October 2012; provisionally accepted 31 March 2013; accepted 25 April 2013)

The aim of studying and analysing vibrational characteristics of structures using power flow is to control the noise and vibration within the structure and prevent it from being transmitted into the environment. The power flow within a cylindrical shell is investigated because of its importance in designing spacecraft and marine structures. In this paper, a four-point power flow technique was used to examine the effects of flexural and shear forces on the total power flow in a cylindrical shell in free-free conditions. To obtain better results, the exponential window was used considering the use of a hammer for the excitation of the structure. The examining of the results and obtained diagrams determined that the effect of shear forces on the total power flow was more than the effect of flexural forces. Moreover, the precision of two-point and four-point techniques was compared.

1. INTRODUCTION

Nowadays, the attention of many researchers and scholars in the field of vibration science is focused on energy reduction entered from a source, which generates noise to a structure and impedes its circulation into the environment. The Power Flow method is deemed proper for the measurement of vibrational energy in the structure. This method is applied as well in determining noise production source position within the structure. Vibration intensity is defined in rigid bodies and its measurement is performed at the level of structure. The reason for this logic comes from the fact that in thin structures, wave propagation within the structure is normally estimated in a definite way by its propagation in the structure.[1]

Several studies have been conducted on the cylindrical shell. One was made by Noisex. He discovered that the peak power flow of a structure happens in its vibrational mode.[2] Fuller and Fahy inspected the free wave propagation in cylindrical shells. They studied the physical interpretation of wave propagation equations in a coupled system.[3] Langley tested the frequency modification effect and changes of material specifications on the power flow.[4] Zhang investigated the input power flow effect on the shells. He used different external forces and frequency during his study.[5] In his investigation, a cylindrical shell filled infinitely with the fluid was considered. Fang proved that the effect of coupling of the fluid and shell on the response is important.[6]

Another problem to be solved was evaluating energy propagation in an arbitrary thin-walled structure yet unsolved.[7] For the purpose of studying the vibrational behaviour of thin shells, many techniques were developed and applied. Arnold and Warburton applied an energy method and used Lagrange's equation as well as Love's first approximation theory. They studied the free vibration of a thin cylindrical shell with freely supported ends.[8] Lam and Loy applied beam functions as the axial modal functions in the Ritz process for examining boundary condition effects on the free vibration characteristics for a cylindrical shell which was layered with nine different boundary conditions.[9]

Zhang and White inspected the power input of a shell due to point force excitation. A good degree of correlation was observed by comparing the driving and transfer points' test acceleration in the frequency range of study with the theoretical results.[10] Merkulov et al. merely tested the point force excitation of an infinite thin-walled cylindrical shell filled with fluid.[11] Energy flow through an arbitrary cross-section of an infinitely long shell was formulated by Sorokin at various circumferential mode numbers. During other studies performed, the inspected issue was "the energy redistribution among several transmission routes in a near-field and the influence of excitation conditions on steady fluctuations of the overall energy flow in a far field."[12]

In a study examining the dynamic characteristics of cylindrical shells, Heckl obtained the input impedance of a simply-supported cylindrical shell; however, the analysis was not covering the influence of the bending stiffness of the shell. His study had restrictions in estimating input power in practical applications.[13] Harari offered a general formula, which assessed the loss of transmitted energy or power based on the structural impedance of finite and semi-infinite cylindrical shells.14 Missaoui et al. performed another study on the free and forced vibrations of a cylindrical shell with a floor partition, according to variational formulation. In this study, the structural coupling was simulated using artificial spring systems.[15] As for describing structural coupling between the shell and plate, Li et al. applied artificial stiffness, presenting a methodology on the structural acoustic coupling characteristics of a cylindrical shell with an internal floor partition.[16]

Wang and Xing investigated the dynamic characteristics of power flow in a coupled plate-cylindrical shell system.[17] Mandal and his colleagues applied the two-transducer method for naturally-orthotropic plates, in order to estimate vibration energy transfer in technically-orthotropic plates. They concluded that the higher the rigidity of the plate, the lower the vibration energy transmission. As the rigidity of the plates increases, the vibration amplitudes of it decrease, thereby reducing acoustic radiation.[18] Zhu et al. have also studied the vibrational power flow of a thin cylindrical shell with a circumferential surface crack. They showed that changes obviously depend on the depth and the location of the crack.[19]

This paper's objective was to inspect the flexural and shear power flow effect on the total power flow using the four-point technique. Moreover, this was then compared with the two-point technique for a cylindrical shell.

2. GOVERNING EQUATIONS

The assumption is that the dimension, d, in the direction of displacement, is noticeably smaller than the wavelength (i.e., $\lambda < 6d$), which allows the thin theory to be applied while being written in terms of the product of forces (Q) and moments (M) with their corresponding normal velocity (ξ) and angular velocity (θ) as[2]

$$I_x = \left\langle Q_x \dot{\xi} \right\rangle_t + \left\langle M_x \dot{\theta}_x \right\rangle_t + \left\langle M_{xy} \dot{\theta}_y \right\rangle_t ; \qquad (1)$$

where the subscripts x and y denote direction and t shows time. The first term of Eq. (1) is the intensity due to shear forces, which can be written by means of the bending stiffness B as[2]

$$I_{x,sf} = -B \left\langle \frac{\partial}{\partial x} \left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} \right) \dot{\xi} \right\rangle_t . \qquad (2)$$

The second term in Eq. (1) is the component due to the bending moment, which can be written as[2]

$$I_{x,bm} = B \left\langle \left(\frac{\partial^2 \xi}{\partial x^2} + \mu \frac{\partial^2 \xi}{\partial y^2} \right) \left(\frac{\partial \xi}{\partial x} \right) \right\rangle_t . \qquad (3)$$

The third term in Eq. (1) is the term for intensity due to the twisting moment, which can be written as[2]

$$I_{x,tm} = B\left(1-\mu\right) \left\langle \frac{\partial^2 \xi}{\partial x \partial y} \frac{\partial \xi}{\partial y} \right\rangle_t . \qquad (4)$$

A finite difference approximation was applied by Linjama to provide a simplified description only by means of the velocity signals in a lateral direction at two points as[20]

$$I_x = +\frac{2\sqrt{Bm'}}{\Delta} \ \mathrm{Im}\{G_{23}\}. \qquad (5)$$

In Eq. (5), above, G_{23} is the cross spectrum between the velocity signals measured in a lateral direction at the two points demonstrated by subscripts, Δ is their space, and m' is the material surface density.

In a similar vein, the third order spatial derivatives of Eqs. (2) and (3) can be estimated by applying finite difference approximations and the measured velocity at four equally spaced co-linear measurement points. The resulting equations can be shown as[20]

$$I_{x,sf} = \frac{B}{2\omega\Delta^3} \left(\mathrm{Im}\{6G_{23} - G_{13} - G_{24} + G_{21} - G_{34}\} \right) \qquad (6)$$

and

$$I_{x,bm} = \frac{B}{2\omega\Delta^3} \left(\mathrm{Im}\{2G_{23} - G_{13} - G_{24} - G_{21} + G_{34}\} \right) ; \qquad (7)$$

where ω is the angular frequency. The imaginary part of the cross-spectrum in Eqs. (6) and (7) can be obtained from the FFT analyser, which is then post-processed using Matlab or Excel. Furthermore, in Eqs. (6) and (7) the direction of the power flow is from point 1 toward point 4, as shown in Fig. 1. The total intensity is the sum of the two components is[20]

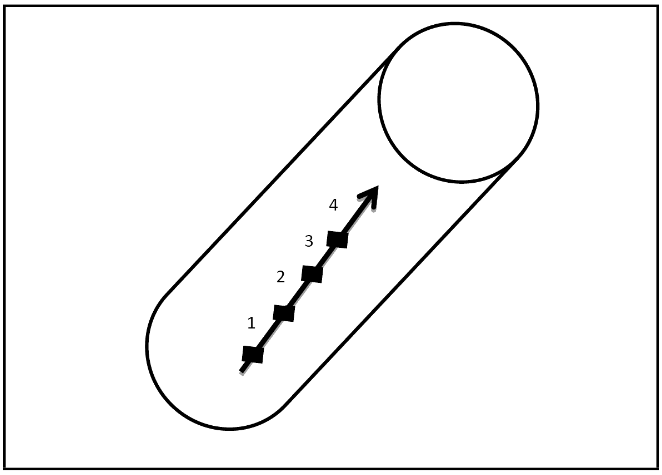

Figure 1. The direction of the flow power in the cylinder.

$$I_x = I_{x,sf} + I_{x,bm} \qquad (8)$$

and

$$I_x = \frac{B}{\omega\Delta^3} \left(\mathrm{Im}\{4G_{23} - G_{13} - G_{24}\} \right) . \qquad (9)$$

For the two-point technique, I_x was calculated from Eq. (5).[20]

3. TECHNICAL CHARACTERISTICS OF THE MEASURED SYSTEM

To measure power flow, a portable vibration analysis device (PVAT) was used, which was produced by Pooya Pajoh Pishro. Also, an impulse force hammer manufactured by Kistler (serial number: 2044659 and model: 724A5000) and an AC 102-A piezoelectric accelerometer sensor manufactured by CTC with the calibration form code of A1-A4 (serial number 81678-81681) were used for measurement.

4. GEOMETRICAL CHARACTERISTICS OF THE MODEL

The selected model was a thin cylindrical shell which was set to be in free-free conditions, as seen in Fig. 2. The length of cylinder was 90 cm with an internal radius and thickness of 16.988 mm and 2 mm, respectively. The cylinder was held using four internal rings with thickness and width of 5 mm and two rings at the two ends with thickness and width of 5 mm and 15 mm, respectively.

5. TEST CONDITIONS AND STIMULATION METHOD

To calculate power flow in the cylindrical shell, the sensors were installed at four points, 20 cm far from each other and in line with the cylinder shaft. After providing the appropriate laboratory conditions, the shell was hung from a firm anchor using an elastic string, in order to provide the free anchor conditions required for conducting the test. Then, the force was applied on the position of Sensor 1 using the hammer and, after doing the necessary adjustments (calculating the spectral multiplication of the domain and phase of the mentioned points), the data were collected in the FFT analyser system in order

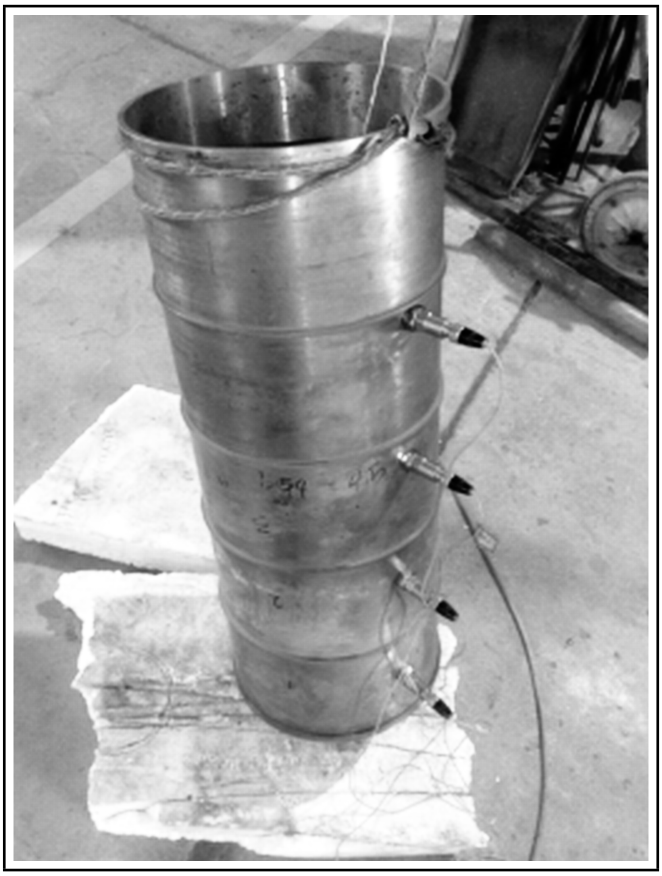

Figure 2. The test rig.

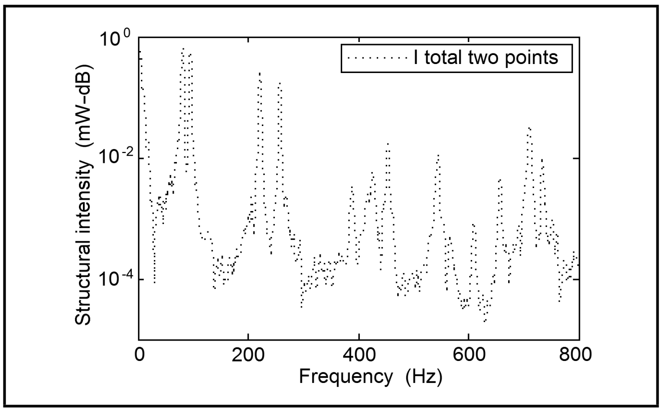

Figure 3. Power flow in the cylinder versus the frequency calculated from the two-point technique.

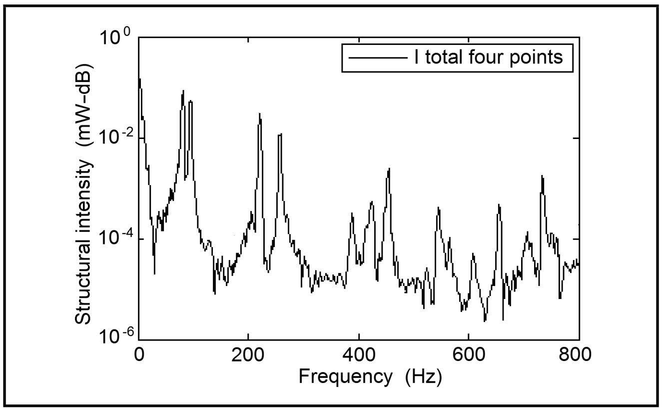

Figure 4. Power flow in the cylinder versus the frequency calculated from the four-point technique.

to calculate the power flow. After that, using algebraic calculations, the amount of power flow in the four-point technique was compared in shear, flexural, and total conditions. Moreover, the information obtained from the four-point technique and the data collected using the two-point technique were analysed.

6. EXPERIMENTAL RESULTS

Figure 3 depicts the power flow of the cylinder versus frequency in the two-point technique. The power flow for the four-point technique is displayed in Fig. 4. In these figures, the power flow is in mW. A comparison between the two figures revealed that the effect of noise on the two-point technique is greater than on the four-point technique. In other words, one can say that the two-point technique is more prone to noise than the four-point technique. Since the two-point technique rank is lower than the four-point technique rank, it has less precision for determining the total amount of power flow. Therefore, as can be observed in Fig. 5, the total amount of power flow is noticeably different in the two-point and four-point techniques.

Figure 6 shows both the flexural and shear power flow of the cylinder. The graph indicates that both power flows are increasing drastically in the vicinity of the natural frequencies. It means that both power flows are increased near the natural frequency and by distancing from it, they both decrease rapidly. As can be seen in Fig. 6, the total amount of power flow propagated in the shell was equal to the shear power flow, and the flexural power flow did not have a large influence on the total amount of power flow. Therefore, it can be concluded that flexural (bending) power flow can be disregarded when calculating power flow. The phase mismatch is an important source of error in both the two-point and four-point approximations. It is well known that there is an optimal spacing between measurement points that makes this error minimum. Also, it is recognized that the optimal spacing is a function of the wavelength. For two accelerometers and a dual channel FFT analyser, earlier studies by Redmon-White[21] have proposed an operating range of $0.15\lambda < \Delta < 0.2\lambda$.

However, a systematic study of point spacing for the two-point and four-point methods has not been made for general cases. For both methods there is a bias and the choice of spacing between measurement points is critical in order to obtain accurate intensity estimations. A very small spacing causes an overestimation, while underestimation will occur if a large spacing between accelerometers is used. Incorrect estimates of the intensity direction are another difficulty for large spacing between measurement points. It should be noted that the four-point method is more sensitive to the spacing between points and can produce large errors when a value outside the optimal range is used. In this study, a trial-error method is used to find out the optimal value for spacing between the four accelerometers.

7. CONCLUSION

The power flow within a cylindrical shell was investigated because of its importance in designing spacecraft and marine structures. A four-point power flow technique was used to examine the effects of flexural and shear forces on the total power flow of the cylindrical shell. To obtain better results, the exponential window was used, considering the use of a hammer for the excitation of the structure. The experimental results indi-

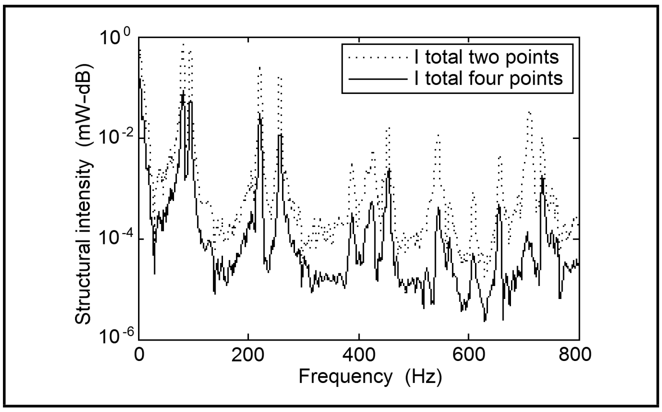

Figure 5. A comparison between power flows calculated from the two-point and four-point techniques.

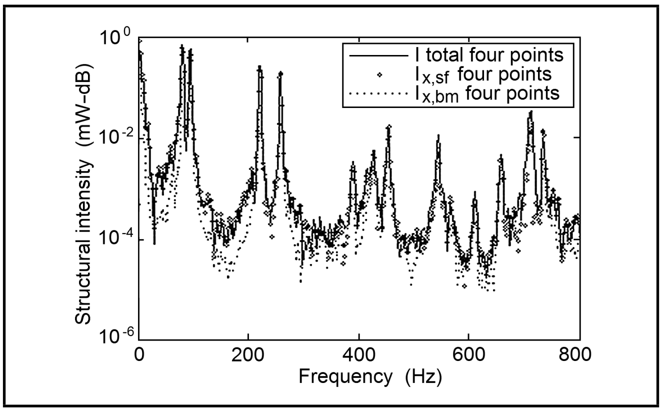

Figure 6. Flexural and shear power flow in the cylinder.

cate that the effect of shear forces on the total power flow was more than the effect of flexural forces. Moreover, the precision of the two-point and four-point techniques was compared with each other. Also, noise affects the results of the two-point technique more than it does the four-point technique.

REFERENCES

1 Pavic, G. Determination of sound power-flow in structures: principles and problems of realization, *Proceeding of International Congress on Recent Developments in Acoustic Intensity Measuring*, CETIM, Senlis, France, 209–215, (1981).

2 Noiseux, D. U., Measurement of power flow in uniform beam and plates, *Journal of the Acoustical Society of America*, **47** (1), 238–247, (1970).

3 Fuller, C. R. and Fahy, F. J. Characteristics of wave propagation and energy distribution in cylindrical elastic shells filled with fluid, *Journal of Sound and Vibration*, **81** (4), 501–518, (1982).

4 Langley, R. S. Wave motion and energy flow in cylindrical shells, *Journal of Sound and Vibration*, **169** (1), 29–42, (1994).

5 Zhang, W. H. and Zhang, X. M. Vibrational power flow in a cylindrical shell with periodic stiffeners, *Proceeding of ASME PVP*, Atlanta, U.S.A, (1991).

6 Feng, L. Acoustic propagates of fluid-filled elastic pipes, *Journal of Sound and Vibration*, **176** (3), 399–413, (1994).

7 Garvic, L. and Pavic, G. A finite element method for computational of structural intensity by the normal mode approach, *Journal of Sound and Vibration*, **164** (1), 29–43, (1993).

8 Arnold, R. N. and Warburton, G. B. Flexural vibrations of the walls of thin cylindrical shells having freely supported ends, *Proceedings of the Royal Society London A086*, 127–145, (1949).

9 Lam, K. Y. and Loy, C. T. Effects of boundary conditions on frequencies of a multi-layered cylindrical shell, *Journal of Sound and Vibration*, **188** (3), 363–384, (1995).

10 Zhang, X. M. and White, R. G. Vibrational power input to a cylindrical shell due to point force excitation, *Proceedings of the 4th International Congress on Intensity Techniques*, France, (1993).

11 Merkulov, V. N., Prikhodko, V. Y., and Tyutekin, V. V. Normal modes in a thin cylindrical elastic shell filled with fluid and driven by forces specified on its surface, *Soviet Physics Acoustics*, **25** (1), 51–54, (1979).

12 Sorokin, S. V., Nielsen, J. B., and Olhoff, N. Green's matrix and the boundary integral equation method for the analysis of vibration and energy flow in cylindrical shells with and without internal fluid loading, *Journal of Sound and Vibration*, **273** (3–5), 815–847, (2004).

13 Heckl, M. Vibrations of point-driven cylindrical shell, *Journal of the Acoustical Society of America*, **34** (10), 1553–1557, (1962).

14 Harari, A. Wave propagation in cylindrical shells with finite regions of structural discontinuity, *Journal of the Acoustical Society of America*, **62** (5), 1196–1205, (1977).

15 Missaoui, J., Cheng, L., and Richard, M. J. Free and forced vibration of a cylindrical shell with a floor partition, *Journal of Sound and Vibration*, **190** (1), 21–40, (1996).

16 Li, D. S., Cheng, L., and Gosselin, C. M. Analysis of structural acoustic coupling of a cylindrical shell with an internal floor partition, *Journal of Sound and Vibration*, **250** (5), 903–921, (2002).

17 Wang, Z. H., Xing, J. T., and Price, W. G. A study of power flow in a coupled plate-cylindrical shell system, *Journal of Sound and Vibration*, **271** (3–5), 863–882, (2004).

18 Mandal, N. K., Rahman, R. A., and Leong, M. S. Experimental investigation of vibration power flow in thin technical orthotropic plates by the method of vibration intensity, *Journal of Sound and Vibration*, **285** (1), 669–695, (2005).

19 Zhu, X., Li, T. Y., Zhao, Y., and Yan, J. Vibrational power flow analysis of thin cylindrical shell with a circumferential surface crack, *Journal of Sound and Vibration*, **302** (1–2), 332–349, (2007).

20 Linjama, J. and Lahti, T. Estimation of bending wave intensity in beams using the frequency response technique, *Journal of Sound and Vibration*, **153** (1), 21–36, (1992).

21 Redmon-White, W. The experimental measurement of flexural wave power flow in structures, *Institute of Sound and Vibration Research*, University of Southampton, 467–474, (1983).