Forced Response Approach to Predict Parametric Vibration

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In this paper, forecast modelling based on modulation feedback is used to investigate the forced response of parametric vibration with a damper. The system is excited by both the periodic coefficient and external force terms, which have different periods. In this study, the forced response is expressed as a linear combination of harmonic components. By applying harmonic balance, the parametric equation is converted into a set of infinite-order linear algebraic equations. Then, by taking the limit to infinity, all coefficients of the harmonic components in the forced response are fully expanded into a series. The advantages of the presented approach are (1) the forced response expressed as a trigonometric series is easier to apply in practice and (2) all coefficients of the harmonic components can be determined by numerical computation. The accuracy of the proposed approach has been verified by comparing resulting phase diagram trajectories with those obtained by the standard Runge-Kutta method. The results show that the presented approach is suitable for the forced response approach and the nonlinear characterization of parametric vibration.

1. INTRODUCTION

The problem with parametric vibration arises in many branches of physics and engineering, and stability and response prediction are the two most significant dynamic problems in the parametric vibration system. In the past, several methods have been used to study the stability of systems with periodic coefficients. These include Hill's method,[1] the perturbation method,[2] the averaging approach, the Floquet theory with numerical integration,[3] and Sinha's numerical scheme with the shifted Chebyshev polynomial,[4,5] etc.

Many approaches, such as the general solution of the Mathieu equation expressed in terms of auxiliary periodical functions,[6] David's transfer matrix method,[7] linear combination of the Floquet eigenvectors,[8] and improved direct spectral method,[9] etc., have been used to find the response expression. Some of these were able to efficiently find the forced response in a multi-degree of freedom system in terms of periodic coefficients. However, the forced response would have more practical value if it were expressed in the form of a Fourier series. For example, in the mechanical fault diagnosis for a rotor with a cross crack, such an expression would be irreplaceable.[10] So far, none of the above mentioned approaches has directly considered using a Fourier series solution.

This paper applies the concept of forecast modelling based on modulation feedback to investigate the forced response. The investigation results in the mathematical derivation of the response in terms of a trigonometric series. Although the concept of forecast modelling has its roots in the modulation system, it can represent the physical characteristics of the parametric vibration system.

2. MODULATION FEEDBACK CONCEPTION

Consider a system excited by both the periodic coefficient and external force terms, which have different periods, as described in Eq. (1):

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2(1+\beta\cos\omega_o t)x = F\cos\omega_p t. \quad (1)$$

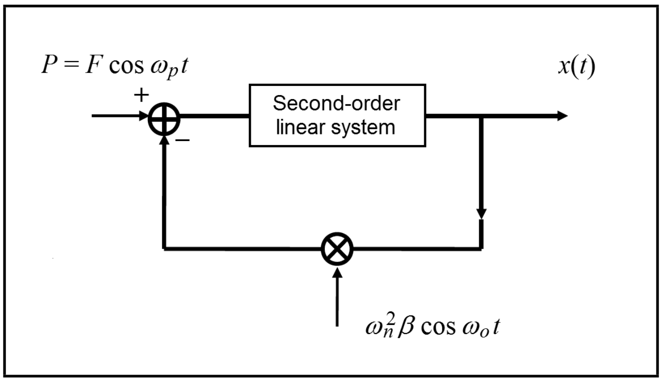

Figure 1. Modulation feedback system.

The above equation can be rewritten as

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2 x = F\cos\omega_p t - x\omega_n^2\beta\cos\omega_o t. \quad (2)$$

Based on Eq. (2), the forced response can be schematically represented in Fig. 1, which depicts a feedback system that contains a second-order linear system and an amplitude modulation. The output of the system is the forced response of the parametric vibration.

Frequency splitting is observed in the system as a result of the amplitude modulation (see Fig. 2). In the first step where $t = 0$, the response of the second-order linear system contains only the frequency component of the excitation force. As a result of the amplitude modulation in the feedback, the frequency spectrum of the signal is split into two new side components: $\omega_p - \omega_o$ and $\omega_p + \omega_o$. The three harmonics now appear as the next input to the second-order linear system. Each of the three components will again excite the same three frequency components at the output of this system. In the second step, three components of the output response signal are again affected by the amplitude modulation in the feedback, each of the new components gets two side components, and the frequency spectrum of the signal at the input of the linear system is expanded again by new components ω_p, $\omega_p - \omega_o$, $\omega_p - 2\omega_o$,