Acoustical Analysis of a General Network of Multi-Port Elements — An Impedance Matrix Approach

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The present work deals with a generalized algorithm for analyzing a network of linear passive acoustic filters having multi-port elements which are interconnected to each other in an arbitrary manner through their respective ports or through general 2-port elements. A multi-port element is characterized by an impedance $[\mathbf{Z}]$ matrix, and the junctions through which these multi-port elements are connected are characterized by conditions of continuity of acoustic pressure and mass velocity. A connectivity matrix is written for the entire network, wherein the interconnections of the elements are taken care of by proper bookkeeping of the acoustic state variables. The acoustic pressures at the external nodes (at the network terminations) are related to mass velocity at the external ports by inversion of the connectivity matrix to obtain the global impedance matrix, characterizing the entire network. This characterization thereby offers a generalized formulation for dealing with a network of multi-port elements. Generalized expressions are obtained for determination of the acoustic performance parameters (Transmission loss (TL), Insertion loss (IL), and Level Difference (LD)) for a multi-port system in terms of the $[\mathbf{Z}]$ matrix and scattering $[\mathbf{S}]$ matrix. A simple method is proposed for evaluation of the $[\mathbf{Z}]$ matrices by means of the axial plane wave theory to characterize long chamber mufflers for a uniform area, conical and exponential duct having an arbitrary number of ports, whilst the ports can be located on the end faces as well as on the side surfaces. The $[\mathbf{Z}]$ matrices characterizing each of these multi-port elements are then used to analyze arbitrary networks of such multi-port elements, and the results are compared with those obtained by 3-D FEA and also against the existing literature.

1. INTRODUCTION

A typical modern day, commercially used, muffler usually consists of a significantly intricate internal structure in which the acoustic elements are interconnected to each other providing multiple connections, and thereby facilitating nonunique paths of wave propagation. The technique of cascading or successively multiplying the transfer $[\mathbf{T}]$ matrices of 2-port elements that are connected to each other sequentially (the outlet of the preceding element is connected to the inlet of the succeeding element) is the most popular method of analyzing mufflers.[1] However, a limitation of this approach is that it is suited only for studying problems in which the direction of the wave propagation can be predetermined; or in other words, the acoustic energy propagation is unidirectional. Such an approach would be useful only if all the elements are connected in series, or if there is no wave propagation path between two nonadjacent elements. This feature is illustrated in Fig. 1. However, in Fig. 2, the acoustic wave propagation direction is nonunique, inasmuch as junction $J2$ offers two alternate paths of wave propagation, thereby making the method of cascading of transfer matrices inapplicable. This particular arrangement, in which the elements are not sequentially connected, but interconnected amongst themselves so as to facilitate a nonunique direction of wave propagation, is called a network arrangement.

One of the earlier works in the analysis of acoustic filters with multiple connections was that of a simple Herschel-Quincke tube due to Stewart.[2] Later, Selamet et al. investigated the same problem, wherein they evaluated the acoustic performance of the Herschel-Quincke tube in terms of Transmission loss (TL) characteristics.[3] Their work removed the restrictions on the duct cross-sectional area, and a general analytical expression for TL was derived, based upon the classical wave equation. Later, Selamet and Easwaran provided a fairly general extension of the two-duct Herschel-Quincke tube problem, wherein an N-duct problem was considered.[4] All the ducts were connected at one junction, and the second port of each of these uniform cross-section ducts met at the other junction, with the lengths of the ducts being unequal, in general. An analytical expression for TL characteristics was obtained for a three-duct configuration based upon the classical 1-D wave propagation equation. Panigrahi and Munjal[5] later extended the above work by Selamet and Easwaran[4] to analyze a general network of 2-port acoustic filters interconnected to each other. They chose to define their analysis of the network of 2-port systems as "generalized multiply connected acoustic filters." They argued that instead of considering only uniform cross-section pipe, a general 2-port network such as an expansion chamber, concentric tube resonators, and extended inlet and outlet systems, could be considered provided their transfer matrix was known *a priori*. Their work was not confined to analyze only modified Herschel-Quincke tube configurations (where the inlets and outlets of all the N ducts were connected together), but in general could analyze a network of 2-port elements connected in an arbitrary manner. However their investigation was limited to an overall network having only a single inlet and single outlet. A general connectivity matrix was written for the entire network, and the inversion of the same provided a final relation (the overall $[\mathbf{T}]$ matrix) between the state variables at the inlet and the outlet.