Vibration Signature Analysis of High-Speed Unbalanced Rotors Supported by Rolling-Element Bearings due to Off-Sized Rolling Elements

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In this paper an analytical model has been developed to investigate the nonlinear dynamic behavior of an unbalanced rotor-bearing system due to ball size variation of the rolling elements. Two cases of ball-size variation were considered: variations of 0.2 micron and 2 microns. In the analytical formulation, the contact between rolling elements and inner/outer races was considered a nonlinear spring, which became stiff using the Hertzian elastic deformation theory. A detailed contact-damping model reflecting the influences of the surface profiles and the speeds of both contacting elements was developed and applied in the rolling-element bearing model. The mathematical formulation accounted for the sources of nonlinearity, such as the Hertzian contact force, varying speed, and radial internal clearance. The equations of motion of a rolling-element bearing were formulated in generalized coordinates, using Lagrange's equations that consider the vibration characteristics of the individual constituents, such as inner race, outer race, rolling elements, and shaft, in order to investigate the structural vibration of the bearing. All results have been presented in form of Fast Fourier Transformations (FFT) and Poincaré maps. The highest radial vibrations due to ball-size variation were at a speed of the number of balls multiplied by the cage speed ($\omega = k\omega_{\text{cage}}$ Hz). The other vibrations due to ball-size variation occurred at $VC \pm k\omega_{\text{cage}}$, where k was a constant. The current study provides a powerful tool for design and health monitoring of machine systems.

NOMENCLATURE

$F_{\text{d-in}}$	– roller-inner race contact damping force
$F_{\text{d-out}}$	– roller-outer race contact damping force
F_{u}	– Unbalanced rotor force, N
k_{in}	– equivalent non-linear contact stiffness of the roller-inner race contact
k_{out}	– equivalent non-linear contact stiffness of the roller-outer race contact
$k_{\text{in-contact}}$	– contact stiffness of the roller-inner race contact
$k_{\text{out-contact}}$	– contact stiffness of the roller-outer race contact
c_{in}	– equivalent viscous damping factor of the roller-inner race contact
c_{out}	– equivalent viscous damping factor of the roller-outer race contact
I	– moment of inertia of each rolling element
I_{cage}	– moment of inertia of the cage
I_{in}	– moment of inertia of the inner race
I_{out}	– moment of inertia of the outer race
M_{in}	– mass of the inner race, kg
M_{j}	– mass of the rolling elements, kg
M_{out}	– mass of the outer race, kg
M_{rotor}	– mass of the rotor, kg
N_{b}	– number of balls
R	– radius of outer race
r	– radius of inner race
r_{in}	– position of mass centre of inner race
r_{out}	– position of mass centre of outer race
T	– kinetic energy of the bearing system
T_{cage}	– kinetic energy of the cage
$T_{\text{i-race}}$	– kinetic energy of the inner race
$T_{\text{o-race}}$	– kinetic energy of the outer race
$T_{\text{r.e}}$	– kinetic energy of the rolling elements
V	– potential energy of the bearing system
V_{cage}	– potential energy of the cage
$V_{\text{i-race}}$	– potential energy of the inner race
$V_{\text{o-race}}$	– potential energy of the outer race
$V_{\text{r.e}}$	– potential energy of the rolling elements
V_{spring}	– potential energy of the springs
$x_{\text{in}}, y_{\text{in}}$	– centre of inner race
$x_{\text{out}}, y_{\text{out}}$	– centre of outer race
$\delta_{\text{in}\,+}$	– contact deformation of the roller-inner race
$\delta_{\text{out}\,+}$	– contact deformation of the roller-outer race
$\left(\dot{\phi}\right)_{\text{in}}$	– angular velocity of inner race
$\left(\dot{\phi}\right)_{\text{in}}$	– angular velocity of outer race
δ	– deformation at the point of contact at inner and outer race, mm
$\Delta\Gamma$	– diameter difference of the off-sized ball, μm
γ	– internal radial clearance
λ	– Lyapunov exponent
ω_{cage}	– angular velocity of the cage, rad/s
ω_{inner}	– angular velocity of the inner race, rad/s
ω_{outer}	– angular velocity of the outer race, rad/s