Influences and simplifications of diffraction functions in nonlinear acoustical parameter measurement systems

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This paper presents an analytical formulation for correcting the diffraction associated with the second harmonic of an acoustic wave, that is more compact than that usually used. This new formulation resulting from an approximation of the correction applied to the fundamentals, which makes it possible to obtain simple solutions for the average second harmonic acoustic pressure but is sufficiently precise when measuring the parameter of nonlinearity B/A in the finite amplitude method. Comparisons with other expressions requiring numerical integration show that the solutions are precise in the near field. Furthermore, the effect of diffraction in the B/A parameter measurement system is discussed.

1. INTRODUCTION

In acoustic parameter measurements of a medium, it is necessary to take into account the diffraction effects of the ultrasonic source to improve the precision of measurements. The measurement cells usually used in transmission consist of two circular transducers (one used as a source and another as a detector). In these situations, the detector will translate into an electric voltage of the average acoustic pressure in its reception area. The analytical solutions describing this average pressure can be formulated as the sum of two terms, one corresponding to the propagation of a plane wave, and the other including the effects of diffraction generated by the geometry of the source-detector unit.

The attenuation α and velocity c can be obtained in the case of linear acoustics. Different authors[1–4] give exact and asymptotic expressions of the average pressure received by a circular transducer. These expressions permit correction functions of diffraction in velocity and attenuation measurements.[5,6]

On the other hand, the B/A parameter is measured in the field of nonlinear acoustics. This parameter is defined as the ratio of coefficients of quadratic term to the linear term in Taylor expansion of the state equation. Consequently, it characterizes the dominant finite-amplitude contribution to the sound speed for an arbitrary fluid.[7] The first measurements of B/A parameter by finite amplitude methods rested on an analytical expression of the second harmonic by considering the propagation of a plane wave.[8–10] Various authors[11,12] then improved the precision of these methods by including a function to correct the diffraction effect resulting from the relation established by Ingenito and Williams[13] for the average pressure exerted by the second harmonic. However, the correction of diffraction obtained is not very practical because it can be evaluated only by numerical integration.

The objective of this paper is to show that one can obtain a simple and precise form by simplifying the correction function of diffraction for the fundamental. Then we will give simple

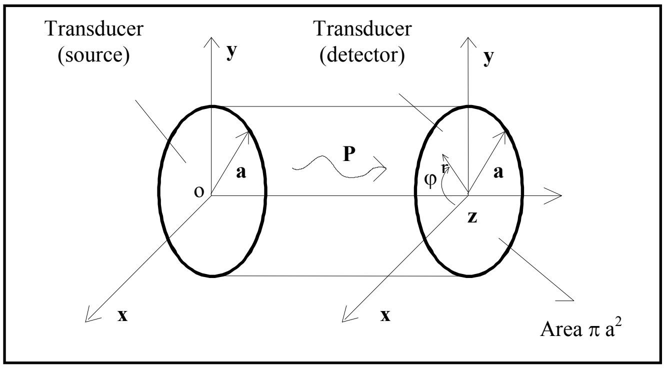

Figure 1: Geometrical configuration of the source-detector.

expressions of the average pressure exerted by the second harmonic, including diffraction and attenuation effects. We will show that the results obtained are equivalent to those established by Coob and validated in measurement systems.[11] But before establishing this result it is necessary to present the various corrections of diffractions applicable to the fundamental from acoustic pressure.

2. CORRECTION OF DIFFRACTION FOR THE FUNDAMENTAL

2.1. Function $D_1(z)$ of diffraction correction for the fundamental

For the nondissipative case ($\alpha_1 = 0$), Williams[1] give the exact expression of the average velocity potential (Fig. 1):

$$\langle \phi_1(r,z)\rangle = \frac{jU_0}{k}e^{jkz} - \frac{j4U_0}{k\pi}\int_0^{\pi/2} e^{jk[z^2+4a^2cos^2\theta]^{1/2}} sin^2\theta d\theta \quad (1)$$

with a being the radius of the transducer.

The first term represents the velocity potential in the case of a plane wave, therefore the average velocity potential on the area of reception is $\phi_{10}(z) = \frac{jU_0}{k} e^{jkz} = \langle \phi_{10}(r, z) \rangle$. The second part of Eq. (1) corresponds to the diffraction effect on the velocity potential, where U_0 is the source amplitude velocity and k is the wave number.

The space average acoustic pressure applied on the receiver is expressed in the form:

$$\langle p_1(r, z) \rangle = -j\rho_0\omega \langle \Phi_1(r, z) \rangle.$$

The correction diffraction function $D_1(z)$ allows to adapt the theoretical plane wave to a real situation. Consequently:

$$D_1(z) = \frac{\langle \phi_1(r, z) \rangle}{\langle \phi_{10}(r, z) \rangle} = \frac{\langle p_1(r, z) \rangle}{\langle p_{10}(r, z) \rangle}$$

with

$$\langle p_{10}(r, z) \rangle = P_0 e^{jkz}. \quad (2)$$

Here $\langle p_{10}(r, z) \rangle$ is the average pressure provided by the fundamental in the case of a plane wave, where $P_0 = \rho_0 c_0 U_0$ is the average acoustic pressure (the source). Thus the modulus of the average pressure is given in dissipative medium in the form:

$$|\langle p_1(r, z) \rangle| = P_0 e^{-\alpha_1 z} |D_1(z)| \quad (3)$$

The exact solution for $D_1(z)$ is obtained from the Williams Eq. (1):

$$D_1(z) = 1 - \frac{4}{\pi} e^{-jkz} \int_0^{\pi/2} e^{jk[z^2 + 4a^2 cos^2\theta]^{1/2}} sin^2\theta d\theta. \quad (4)$$

2.2. Simplifications of the function of correction $D_1(z)$

For $z > a$ Bass[2] gives a good approximation for Eq. (4) which can be simplified for $\xi(z) \gg 1$ in the form:

$$D_1(z) \approx 1 - \left(1 - \frac{\xi(z)^2}{2(ka)^2}\right) \left(\frac{2}{\pi\xi(z)}\right)^{1/2} e^{-j\pi/4} \quad (5)$$

with $\xi(z) = \frac{k}{2}\sqrt{z^2 + 4a^2} - z$. This expression was used by Coob[10] to reduce $D_1^2(z)$ and to evaluate the average pressure of the second harmonic.

By limiting the development of $[]^{1/2}$ to the first order in Eq. (4), Rogers et al.[4] obtain the following good approximation:

$$D_1(z) \approx 1 - e^{-j\frac{ka^2}{z}} \left[J_0\left(\frac{ka^2}{z}\right) + jJ_1\left(\frac{ka^2}{z}\right)\right], \quad (6)$$

where J_0 and J_1 are the Bessel functions of the fist kind of orders 0 and 1 respectively.

Equation (6) is valid for all the values of z/a if $(ka)^{1/2} \gg 1$. Compared to the exact Eq. (4) the error using this simplification is lower than 0.4% for $ka = 100$. Considering $z/a < (ka)^{1/2}$, implies that $ka^2/z > (ka)^{1/2} \gg 1$. Thus, one can simplify Eq. (6) by using the asymptotic developments of the Bessel functions:

$$D_1(z) \approx 1 - \left(\frac{2z}{\pi ka^2}\right)^{1/2} e^{-j\pi/4} = 1 - g(z), \quad (7)$$

where $g(z)$ is the diffraction function related to the parameters of source a and k with plane wave case property $\lim_{ka\to\infty} [g(z)] = 0$.

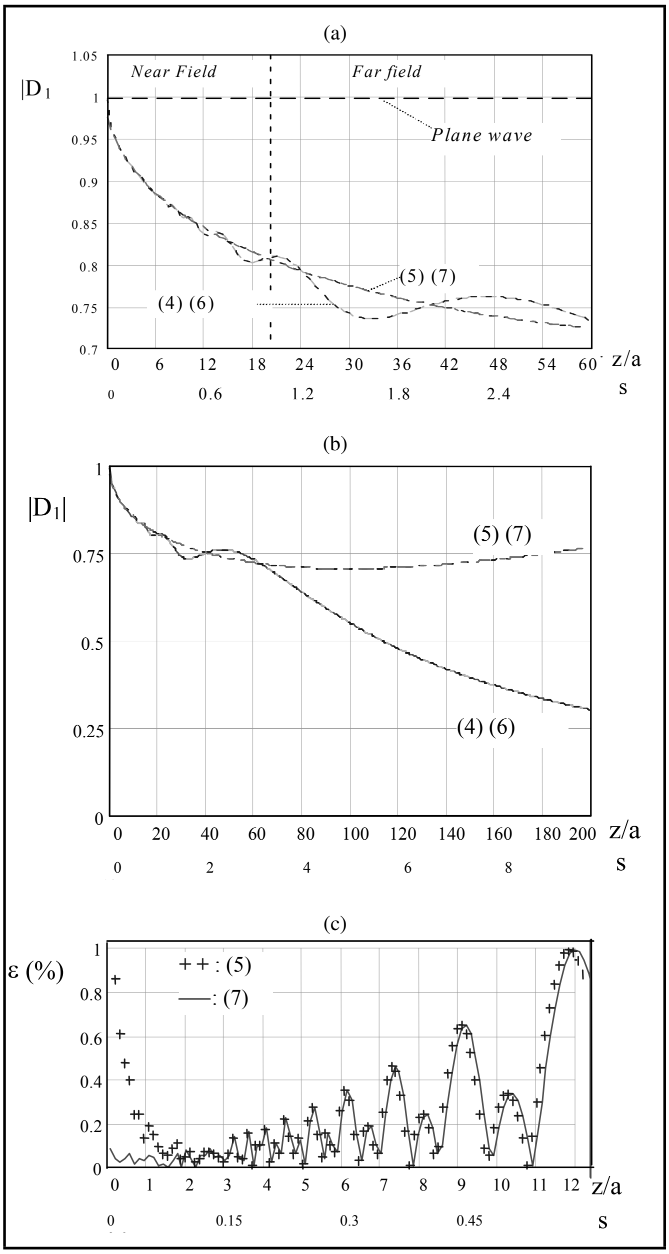

Figure 2: Functions of diffraction correction. Comparison with the exact solution to Williams, Eq. (4). c) presents the relative errors between exact Eq. (4) and Eqs. (5) and (7).

2.3. Comparison of the various expressions of $D_1(z)$

Figure 2a and 2b represent the modulus $|D_1|$ of the different equations. For the y axis, we use two variables z/a and $s = z\lambda/a^2 = 2\pi z/ka^2$ (*The Fresnel number for focused acoustic beams is* $N = 1/s$).[13] With the variable s, we can distinguish the near field ($s \leq 1$) and the far field ($s > 1$). Simulations are obtained with $a = 1$ cm and $ka = 125$.

The curves obtained with simplification of of Eq. (6) and the exact Eq. (4) are superposed, and the asymptotic Eq. (7) then constitutes a good approximation in the near field (Fig. 2a). These equations diverge from the exact solution for $z/a > 60 \quad (s > 3)$ (Fig. 2b). The relative error (Fig. 2c) confirms the range of validity $z/a < (ka)^{1/2} \quad (s < 2\pi/(ka)^{1/2})$ for $(ka)^{1/2} \gg 1$. Thus the relative error is lower than 0.7%. The lower limit is, in any event, limited in experiments to the appearance of standing waves in the measuring cell.

3. CORRECTION OF DIFFRACTION OF THE SECOND HARMONIC

3.1. Function of diffraction correction for the second harmonic

The equation obtained by Ingenito and Williams[13] for the second harmonic in the case of monochromatic waves in non-dissipative media (which can be found in[11]) is a good approximation for this Eq. that can be used in the dissipative case.

The average potential ϕ_2 is given by:

$$\langle\phi_2(r,z)\rangle \approx -\frac{\beta k^2}{4c_0}\cdot \int_0^z e^{jk\psi}e^{-\alpha_2\psi}\left\langle\phi_1^2\left(r,z-\frac{\psi}{2}\right)\right\rangle e^{-2\alpha_1(z-\psi)}d\psi, \quad (8)$$

where $\beta = 1+\frac{1}{2}B/A$ and α_2 is the second harmonic attenuation, $p_2 = -2j\rho_0\omega\phi_2$, and B/A is the parameter of nonlinearity.

Equation (8) is the reference analytical solution for the second harmonic average velocity potential in a dissipative medium. Ingenito and Williams[13] show that a good approximation consists in replacing $\langle\phi_1^2\rangle$ by $\langle\phi_1\rangle^2$ in the expression for $\langle\phi_2\rangle$. Thus, we can write:

$$\begin{aligned}\langle\phi_1(r,z)\rangle^2 &= \langle\phi_{10}(r,z)\rangle^2\left[1-\left\{2g(z)-g(z)^2\right\}\right]\\ &= \langle\phi_{10}(r,z)\rangle^2\left[1-f(z)\right] \qquad (9)\end{aligned}$$

with

$$f(z) = 2g(z)-g(z)^2 = 1-D_1(z)^2. \qquad (10)$$

The average pressure of the second harmonic according to $D_1(z)$ is obtained by using these relations:

$$\langle p_2(r,z)\rangle \approx \left(KP_0^2e^{-2\alpha_1 z}\int_0^z e^{(2\alpha_1-\alpha_2)\psi}D_1\left(z-\frac{\psi}{2}\right)^2 d\psi\right)e^{j2kz} \qquad (11)$$

with $K = \frac{(2+B/A)\omega}{4\rho_0 c_0^3}$.

The function of diffraction in the second harmonic can be given by $D_2(z) = \frac{\langle p_2(r,z)\rangle}{\langle p_{20}(r,z)\rangle}$. Thus, while considering $D_2(z)$ independent of the attenuation, which amounts to separating the effects of the attenuation and diffraction, we obtain:

$$\begin{aligned}D_2(z) &= 1-\frac{1}{z}\int_0^z f\left(z-\frac{\psi}{2}\right)d\psi\\ &= \frac{1}{z}\int_0^z D_1\left(z-\frac{\psi}{2}\right)^2 d\psi. \qquad (12)\end{aligned}$$

3.2. Simplifications of $D_2(z)$ and $\langle p_2(r,z)\rangle$

According to Eq. (12) the correction $D_2(z)$ is related to $D_1^2(z)$, which can be simplified. Since $D_1^2(z) = 1-2g(z)+g^2(z)$ and $\lim\limits_{ka\to\infty}[g(z)] = 0$, we can neglect the term $g^2(z)$ for a large value of ka. Thus Eqs. (7) and (5) with the following conditions: $z/a < (ka)^{1/2}$ and $(ka)^{1/2} \gg 1$ result in this simplified expression:

$$D_1^2(z) \approx 1-2\left(\frac{2z}{\pi ka^2}\right)^{1/2}e^{-j\pi/4} \qquad (13)$$

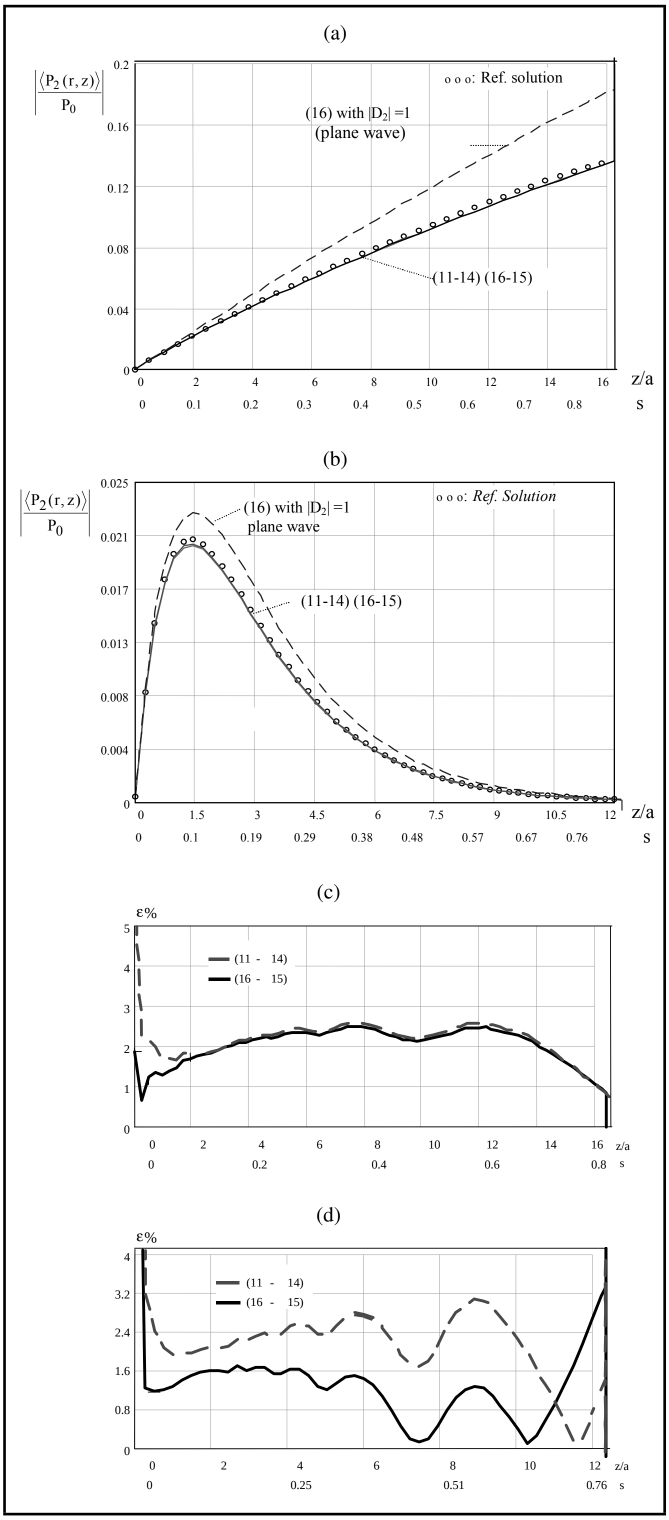

Figure 3: Simulations of analytic solutions of the second harmonic average pressure for water (a) and glycerol (b). Relative variation between reference solution and solutions (15–16), (11–14) for water (c) and glycerol (d).

$$D_1^2(z) \approx 1-2\left(1-\frac{\xi(z)^2}{2(ka)^2}\right)\left(\frac{2}{\pi\xi(z)}\right)^{1/2}e^{-j\pi/4} \qquad (14)$$

We can thus take advantage of the simpler Eq. (13) to calculate a diffraction function $D_2(z)$. In this case, the integration of Eq. (12) results in:

$$D_2(z) \cong 1-C\sqrt{\frac{z}{ka^2}}e^{-j\pi/4}$$

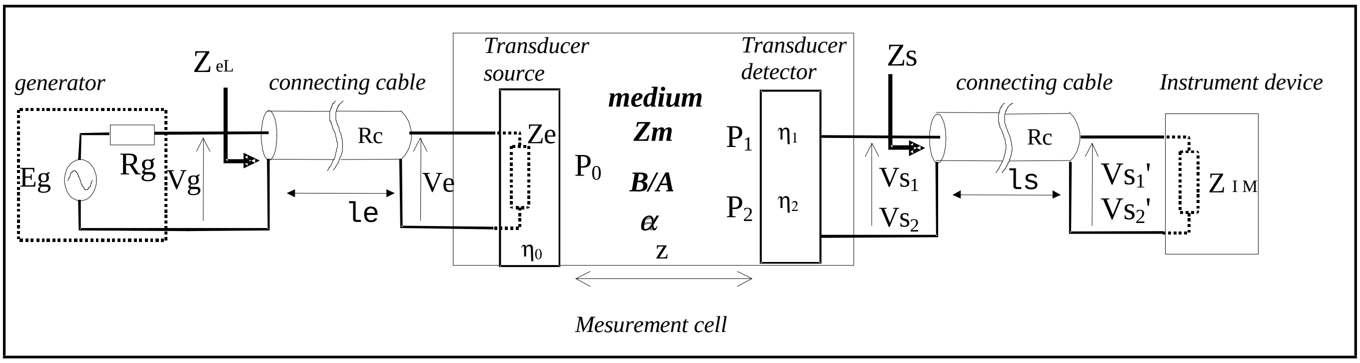

Figure 4: System of B/A parameter measurement by the comparative method.

with

$$C = \frac{4}{3\sqrt{\pi}}(2\sqrt{2} - 1) \approx 1.375. \quad (15)$$

Finally, with Eq. (11), we have established a simple expression that is sufficiently accurate and one that can give the average pressure provided by the second harmonic on a receiver with the same dimensions of the source:

$$|\langle p_2(r, z)\rangle| \approx KP_0^2 \left(\frac{e^{-\alpha_2 z} - e^{-2\alpha_1 z}}{2\alpha_1 - \alpha_2}\right) |D_2(z)|. \quad (16)$$

3.3. Comparison of solutions for the average pressure $|\langle p_2(r, z)\rangle|$

We simulated the expressions of the relative average pressure $|\langle p_2(r, z)\rangle|/P_0$ in two extreme mediums in term of attenuation and nonlinear effects:
Water: $c_0 = 1483$ m/s, $\rho_0 = 1000$ kg/m^3, $\alpha_0 = 0.25 \cdot 10^{-13}$ Npm^{-1}Hz^{-2}, $\alpha_2 = 4 \cdot \alpha_1$, $B/A = 5.2$
Glycerol: $c_0 = 1909$ m/s, $\rho_0 = 1260$ kg/m^3, $\alpha_0 = 26 \cdot 10^{-13}$ Npm^{-1}Hz^{-2}, $\alpha_2 = 4 \cdot \alpha_1$, $B/A = 9.4$
The conditions, close to the Coob experiments are: $f = 3$ MHz, $a = 1$ cm, $I_0 = 0.5$ W/cm^2 for water and $I_0 = 10$ W/cm^2 for glycerol, with $I_0 = P_0^2/(2\rho_0 c_0)$.

The results are presented in Fig. 3. Note that our Eq. (14–15) is similar with that obtained by Coob.[11] The importance of the diffraction correction $D_2(z)$ is visualized by the representation of the simple case of a plane wave, i.e., for $D_2(z) = 1$. We also simulated the relative average pressure obtained with the reference solution (8) and the King integral[15,16] for fundamental ϕ_1.

Relative errors between the reference solution and the solutions (14-15) and (11–14) are presented Fig. 3 c–d for water and glycerol, where simulations are carried out with a tolerance of 10^{-6} for the calculation of the integrals (Romberg Method) with the Mathcad® software. They show that the solution (15–16) is, overall, more precise than Eq. (11–14) and is under the conditions adopted for simulation. Moreover, the computing time necessary for Eqs. (15–16) is better than that of the referenced equations, which includes a triple integral.

4. SIMULATIONS AND ANALYSES OF DIFFRACTION FUNCTIONS

4.1. Measurement system

The objective of this section is to show the effects of diffraction in the B/A parameter measurement system. The ideal model is established on the basis of geometrical configuration, which is presented in Fig. 1. The two transducers are circular, of the same diameter, and spaced to a distance of z. The electric excitation Ve of the transducer source which is supposed to be permanent and have a sinusoidal frequency f_1. The detecting transducer converts the average pressure $P(z, t)$ received on its front face in the electric signal $Vs(z, t)$ of spectral components (Vs_1, f_1) and (Vs_2, f_2). The propagation medium is defined by its density ρ_0, the propagation velocity of the acoustic wave c_0, its nonlinear B/A parameter, and its attenuations α_1 and α_2 at the frequencies f_1 and $f_2 = 2f_1$. The expression of fundamental (Vs_1, f_1) will be established within the framework of linear acoustics and that of the second harmonic (Vs_2, f_2) will be defined using the quasi-linear approximation of the equation of propagation acoustic wave of pressure.[16,17] The average pressure P_0, on which the second harmonic P_2 depends, can be deduced from the excitation V_0 or the measurement of the component Vs_1 from the fundamental detected. We will study this last procedure. Figure 4 describes the functional elements of the measurement system.

4.2. Procedure by measurement of the components Vs_2 and Vs_1

The average acoustic pressures, fundamental P_1 and the second harmonic P_2, can be expressed in the form:[14,18]

$$P_1(z, f) = P_0 e^{-\alpha_1(f)z}|D_1(z, f)| \quad (17)$$

$$P_2(z, f) = K(f)P_0^2 \left(\frac{e^{-\alpha_2(f)z} - e^{-2\alpha_1(f)z}}{2\alpha_1(f) - \alpha_2(f)}\right) |D_2(z, f)|, \quad (18)$$

where P_0 is the average pressure on the transducer. The nonlinear B/A parameter of the medium appears in the term:

$$K(f) = \frac{\pi f(2 + B/A)}{2\rho_0 c_0^3} \quad (19)$$

$\alpha_n(f) = \alpha_0(nf)^q$ is the attenuation at the frequency nf, α_0 and q are the specific coefficients of the medium. D_1 and D_2 are related, respectively, to correct the diffraction of the fundamental (f) and the second harmonic $(2f)$. Under the condition $z/a < (ka)^{1/2}$ for $(ka)^{1/2} \gg 1$, where $k = 2\pi f/c_0$, we obtain good approximations for these functions. The electric quantities are connected to the pressures by:[19,20]

$$V_{s1} = \eta_1 P_1, V_{s2} = \eta_2 P_2, P_0 = \eta_1 V_0, \quad (20)$$

where η_1, η_2: is the sensitivity of the transducer in reception (detector) for 1st and 2nd harmonic, and η_0 is the sensitivity of the transducer in emission (source).

With the preceding relations and by applying the procedure of the simple comparative method, we can establish the complete expression for the parameter B/A:

$$\left(\frac{B}{A}\right)_x = \frac{Vs_{2_x}}{Vs_{2_r}}\left(\frac{Vs_{1_r}}{Vs_{1_x}}\right)^2 F\eta\left\{FD_{iff}F\alpha\right\}\cdot$$
$$\cdot\frac{\rho_x c_x^3}{\rho_r c_r^3}\left[\left(\frac{B}{A}\right)_r + 2\right] - 2, \quad (21)$$

where we define:

$$F\eta = \left|\frac{\eta_{2_r}}{\eta_{1_r}^2}\frac{\eta_{1_x}^2}{\eta_{2_x}}\right|; F\alpha = \frac{F(z_r, \alpha_{1_r}, \alpha_{2_r})}{F(z_x, \alpha_{1_x}, \alpha_{2_x})} \quad (22)$$

$$FD_{iff} = \left|\frac{D_2(z_r, a, k_r)}{D_2(z_x, a, k_x)}\right|\left|\frac{D_1(z_x, a, k_x)^2}{D_1(z_r, a, k_r)^2}\right|. \quad (23)$$

F_η is the sensitivity function of the measurement system. F_α represents the attenuation function and FD_{iff} takes into account the diffraction effects for the fundamental and the second harmonic. Indices r and x indicate the reference medium and the medium under investigation, respectively. Simplified expressions of B/A parameter can be obtained with approximations on the three functions defined previously, F_η, F_α and FD_{iff}. They lead to $FD_{iff} = 1$ if the influence of diffraction is neglected, $F_\alpha = z_r/z_x$ if the influence of attenuation is neglected and $F_\eta = 1$ if the influence of the transducer sensitivities are neglected. All these approximations will be made with the disadvantage of precision in B/A parameter measurements.

We want to analyze the effect of the diffraction function FD_{iff} on the B/A parameter measurement in a comparative method, FD_{iff} is given by Eq. (23).

The transducers shown in Fig. 4 will be fixed, therefore $z = z_r = z_x$. For the correction D_2 one can use the simplified form (Eq. 15). For the D_1 correction, one must choose Eq. (6), which is a simplification of the Bass Eq. 2 and practically equivalent to the exact solution of Williams Eq. (4). A simple asymptote of this correction is given by relation (5). Simulations are carried out at the resonance frequency of the source transducer (2.02 MHz) by taking the relative position z/a variable. Water is the reference medium ($c_r = 1500$ m/s), ethanol ($c_x = 1158$ m/s) and glycerol ($c_x = 1900$ m/s) are the analysed mediums. The results obtained are presented in Fig. 5.

Note that the asymptote obtained with D_1 (Eq. 4) is only valid in the very close domain ($z/a < 5$), the amplitude of the undulations of the exact solution increases quickly beyond that. Moreover, in this zone, the diffraction function can be neglected ($FD_{iff} = 1$) without degrading the measurement accuracy of the B/A parameter. One can define the error brought to the measurement of the B/A parameter, if the influence of the diffraction is neglected, in the form:[21]

$$\varepsilon_{B/A} = \left|1 - FD_{iff}^{-1}\right| \cdot \left[1 + 2\cdot(B/A)^{-1}\right] \quad (24)$$

This error, presented in Fig. 5b, is more significant when:

1. the detector transducer is far from the source transducer, and

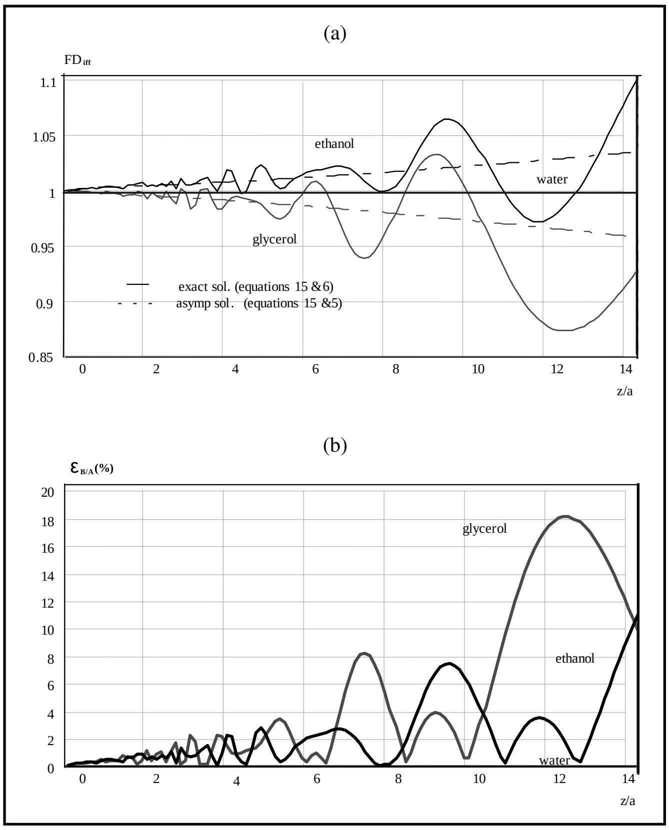

Figure 5: Diffraction Function and associated error $\varepsilon_{B/A}$ for $f_1 = fr_S = 2.02 MHz$. ($ka_{ethanol} \approx 68$, $ka_{glycerol} \approx 53.5$).

2. the wavelength in the analyzed medium is different from that in the reference medium.

The validity zone of the diffraction function is limited by that of the D_2 function ($2 < z/a \ll k \cdot a$).

5. CONCLUSION

We have shown in this paper that we can obtain a function of diffraction correction for the second harmonic much simpler than those usually used. This new formulation is obtained from a simplification of the correction applied to the fundamental acoustic pressure.

We can use this new and simple expression to describe accurately the second harmonic pressure detected by a transducer. It can be exploited in measurements of the non-linearity parameter B/A. Another interesting aspect of these simple analytical solutions is the significant reduction in the computing times when they are used in simulation processes of systems working in the field of non-linear acoustics. Moreover, we have obtained,for a measurement system for the B/A parameter, the conditions necessary to neglect the diffraction effect.

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