
Identification and Active Feedback-Feedforward Control of Rotor

Kari M. J. Tammi

VTT - Technical Research Centre of Finland, P.O. Box 1000, FI-02044 VTT, Finland

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This experimental work presented demonstrates the use of identification, feedback, and feedforward methods to control rotor vibrations. The experiments were performed on a rotor test rig having a 3-kg rotor supported by journal bearings; the first bending resonance of the rotor shaft was about 50 Hz. Identification was carried out with a method taking into account the disturbances due to rotation. The method, using a reference signal generated from speed measurement, was able to discard the forced vibrations due to the mass imbalance. The active control objective was to reduce the radial response at the rotor midpoint by an electromagnetic actuator located outside the bearing span of the rotor. The feedback system was a proportional-derivative-type controller, which increased the damping of the system. A feedforward control system was constructed to work together with the feedback controller. The control methods produced a significant decrease in the midpoint responses of the rotor at sub-critical speeds. For supercritical speeds, the decrease in the responses was more modest due to the restricted control authority. The stability of the feedforward controller was studied in order to explore the relationship between the system damping and the modelling accuracy required by the feedforward control system.

1. INTRODUCTION

This research was carried out in order to study the use of a supplementary actuator for rotor vibration control. The general aim was to explore vibration control solutions for rotor systems with conventional bearings, not to carry the rotor mass by the actuator. The objective of this study was to control the vibrations of the rotor in the test rig by using an electromagnetic actuator. The vibrations were to be attenuated inside the bearing span of the rotor while the actuator was located outside the bearing span. The aim was also to identify the dynamic rotor system by using the data acquired during its operation. Even the test environment used was relatively small; the intention was to expand the principle to larger prototypes and applications to control resonance vibrations of heavy rotors with a supplementary actuator. Research on similar set-ups has been carried out by Cheung et al.,¹ and Ishimatsu et al.²

The test environment had a 560 mm long slim shaft with three disks attached (Figs. 1 and 2). The diameter of the shaft was 10 mm. The total weight of the rotor including the shaft and the disks was 2.7 kg. The rotor was supported by journal bearings whose mutual distance was 360 mm. The rotor was driven by an electrical motor by means of a flexible coupling. Radial displacement sensors were placed at two locations along the shaft: at the midpoint and at the end of the rotor (S1 and S2 in Fig. 1). The control system design consisted of two topologies: feedback control to provide basic damping over a wider frequency band, and feedforward control to compensate the disturbance due to the rotor mass unbalance. The feedforward control algorithm used the midpoint displacement sensors and the feedback algorithm used the endpoint displacement sensors to provide the control error signal. The speed of rotation was measured with a pulse sensor at the drive end of the rotor. The rotation speed measurement was used for generating the reference signal for identification and

feedforward control. Also, the speed measurement was used by the feedforward controller in order to estimate the system frequency response at the speed of rotation. The journal bearings, the actuator, the displacement sensors, and the driving motor were all fixed to a stiff foundation considered ideally rigid in this study.

The electromagnetic actuator was used to produce the control forces at the non-drive end of the rotor. The actuator and its control unit were modified from active magnetic bearing equipment. Being a non-contact device, the actuator exerted the control forces through an air gap on the armature fixed to the rotor (Disk 3 in Fig. 1). The control unit had its own internal force control loop to make the force generation more accurate. The forces exerted were controlled by means of feedback from the magnetic flux density sensors in the air gap between the rotor and the actuator. This control loop was not subjected to any research in this work. The actuator, the control unit hardware, and its internal force control loop were assumed to be "sufficiently fast" for the purpose. The design specification of the actuator was to be capable to produce a force of 500 N up to 500 Hz, whereas the control action took place in the frequency band from 0 to 150 Hz and only a few newtons were required. The forces used were low, in the order of one third of the rotor weight. Thus, the actuator was oversized for this particular purpose. This was due to technical limitations in scaling the system down and to the availability of ready-made magnetic bearing technology. The components of the set-up and their modifications from the original purposes have been reported in more detail by Tammi.³ In contrast to magnetic bearings, the objective was not to have the actuator carry static loads, as the rotor was supported by the journal bearings.

The main contributions of this paper are 1) to present an identification method that automatically discards disturbances correlated with a reference signal, and 2) to demonstrate the roles of feedback and feedforward control algorithms in rotor vibration control. The identification method is

a partial result from an effort to develop automated procedures for rotor system identification and controller tuning. The identification method presented is used to compensate forced vibration responses. In the work, the standard identification problem is formulated in such a way that non-measurable excitations could be included in the problem. Discussion about the roles of feedback and feedforward controllers is related to the studies on the required model accuracy in feedforward compensation systems. The results suggest that a simple feedback system is useful in terms of feedforward system robustness. The stability requirements for adaptive feedforward controllers are commonly known. However, the implication of this study is to point out the relationship between the system damping and the required modelling accuracy to feedforward control.

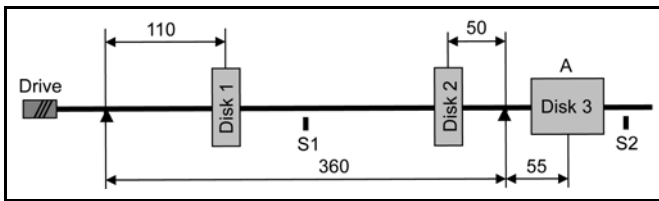


Figure 1. The rotor layout: the displacement sensors at “S1” and “S2” and the actuator at “A”. The dimensions are in millimetres.

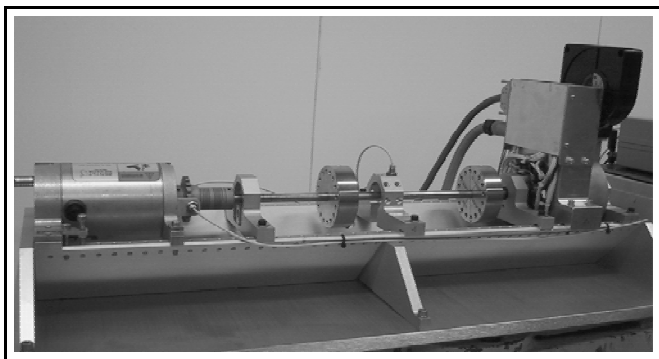


Figure 2. The driving motor (left), the rotor with the disks, and the actuator (right).

The motivation for the work arose from the tightening performance and durability requirements of machines. Improved control of the resonance behaviour in the critical speed region can help in meeting the requirements. The critical speed is the rotation frequency corresponding to the rotor’s first bending resonance frequency. Especially, heavy rotors with conventional journal or rolling element bearings are often designed to work in the sub-critical range. Their maximum operation speed may be limited to less than one third or one half of the critical speed. The reason for limiting the speed is avoidance of an excessive dynamic response that can reduce the process quality, shorten the life of components, or cause disturbances in the environment of the machine. An active control method to reduce rotor responses provides a possibility to increase the machine speed (capacity) and still meet specifications. On the other hand, active control may also allow lighter constructions or cheaper components.

2. MATERIALS AND METHODS

2.1. Identification

The run-time identification of a rotor system is often desirable, because the dynamic behaviour of a rotor may be de-

pendent on its rotation speed and operating conditions. An ordinary identification procedure based on input-output measurements can be deluded by excitations that are not observable in the system input. In rotordynamics, rotation harmonics cause excitations of this kind. These rotation harmonics appear at the frequency of rotation and its multiples (or sometimes submultiples), being caused by rotor unbalance, alignment errors, asymmetry in the rotor or in its bearings etc. (see e.g. Vance⁴). Discarding these excitations before identification usually requires intelligent judgement. For example, the operator can exclude the harmonics before carrying out the modal analysis, or filter those peaks out during the frequency response function measurement. This paper presents a simple method for compensating such peaks, assuming that a suitable reference signal is available. The method is studied in order to reduce the need of human intervention in identification, and to make the identification procedure automatic. Automatic identification is important in order to develop adaptive vibration control systems for rotors. Similar methods have been considered for secondary-path estimation in active noise control (see, for example Kuo and Morgan;⁵ and Hansen and Snyder⁶). These estimation methods took the periodic signal, fed into the system, into account in a manner similar to that presented here. Eriksson et al. compared off-line and on-line estimation methods; the on-line estimation was considered complex and computationally heavy in the work.⁷ Meurers and Veres considered the implementation issues of a computationally light on-line secondary-path estimation method.⁸ In their study, the computationally light method meant avoiding spectral analysis in the disturbance estimation. Bao, Sas, and Brussel compared identification and excitation methods that minimally perturb the plant to be controlled.⁹

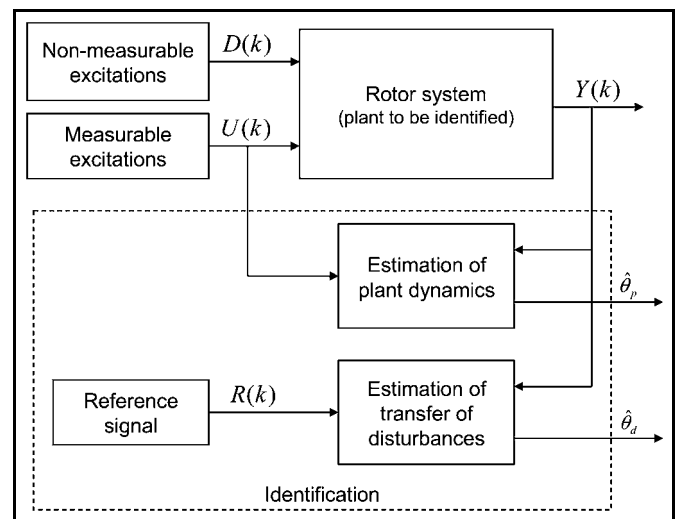


Figure 3. The rotor system to be identified comprised two dynamic systems, one for the actual dynamics, and the other for the disturbance.

The identification scheme studied is shown in Fig. 3. The system to be identified is excited by measurable excitations, $U(k)$, and unknown disturbances (non-measurable excitations, $D(k)$). The response, $Y(k)$, is a sum response of these two excitation components. Carrying out the identification from $U(k)$ to $Y(k)$ can result in the dynamic system having a spurious resonance peak at the frequencies of $D(k)$; this is the problem to be overcome by the present work. In order to accomplish

this, the rotor system is considered as two systems to be identified: 1) the dynamic system from measurable excitations to the output, and 2) the dynamic system from the reference signal to the output. The former describes the dynamics of the actual plant. The latter describes the transfer of the forced vibrations due to rotation. Since the forced vibrations cannot be measured for use as the input signal, it is replaced by a signal correlating with the forced vibrations. The replacing signal used is the reference signal, $R(k)$, generated by means of the rotation speed measurement.

In identification terminology, the rotation harmonics described above are interpreted as load disturbances unobservable in the input of the system.¹⁰ This paper investigates a simple idea for taking these disturbances into account and mitigating their influence on the identification result. The author was inspired by the textbook of Maciejowski, in which an example of predictive control showed how to compensate the constant load disturbance in the output of a system.¹¹ Consider a strictly proper dynamic system in the discrete state-space form

$$X(k+1) = A_p X(k) + B_p U(k); \quad Y(k) = C_p X(k), \quad (1)$$

where $X(k)$ is the state vector, whose length corresponds to the system order, k is an integer representing the sample index. $U(k)$ is the input vector, whose length equals the number of inputs. $Y(k)$ is the measured plant output(s). Matrices A_p , B_p , and C_p with obvious dimensions describe the plant dynamics (i.e. the dynamic system to be identified). A constant load disturbance may be described by means of the augmentation

$$\begin{aligned} \begin{bmatrix} X(k+1) \\ D(k+1) \end{bmatrix} &= \begin{bmatrix} A_p & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} X(k) \\ D(k) \end{bmatrix} + \begin{bmatrix} B_p \\ 0 \end{bmatrix} U(k); \\ Y(k) &= \begin{bmatrix} C_p & I \end{bmatrix} \begin{bmatrix} X(k) \\ D(k) \end{bmatrix}, \end{aligned} \quad (2)$$

where the disturbance $D(k)$ affects the output of the system.¹¹

In order to compensate a disturbance, the Internal Model Principle requires that this disturbance be represented in the compensating system. This means taking the disturbance into account by either modelling it, or by feeding an external compensating signal into the compensating system.¹² Feeding an external compensating signal means using a reference signal correlated with the disturbance. The use of the reference signal seemed a natural solution for a rotor system, since the speed of rotation is often measured and the disturbances occur at the rotation frequency and its multiples. Furthermore, the same reference signal is used for the feedforward compensation. The augmented system with supplementary input equals

$$\begin{aligned} \begin{bmatrix} X(k+1) \\ D(k+1) \end{bmatrix} &= \begin{bmatrix} A_p & 0 \\ 0 & A_d \end{bmatrix} \begin{bmatrix} X(k) \\ D(k) \end{bmatrix} + \begin{bmatrix} B_p & 0 \\ 0 & B_d \end{bmatrix} \begin{bmatrix} U(k) \\ R(k) \end{bmatrix}; \\ Y(k) &= \begin{bmatrix} C_p & C_d \end{bmatrix} \begin{bmatrix} X(k) \\ D(k) \end{bmatrix}, \end{aligned} \quad (3)$$

where $R(k)$ is the reference signal correlating with the disturbance. Matrices A_d , B_d , and C_d filter the reference signal in

such a way that it has the correct effect on the system output. Note again that the reference signal must correlate with the forced vibrations.

Identification was carried out by using the disturbance compensation scheme presented above. The equations were modified into the form required in the standard least squares identification method. Consider a system with single output and single input

$$y(k) = \varphi^T(k) \theta_0, \quad (4)$$

where $\varphi(k)$ is the vector of regression variables at time instant k , and θ_0 is the vector containing the parameter estimates describing the system dynamics.¹³ An infinite-impulse-response (IIR) filter can be expressed as

$$\begin{aligned} \varphi^T(k) &= \left[-y(k-1) \quad -y(k-2) \quad \cdots \quad -y(k-n) \quad \rightarrow \right. \\ &\left. \rightarrow \quad u(k+m-n) \quad u(k+m-n-1) \quad \cdots \quad u(k-n) \right] \end{aligned} \quad (5)$$

and

$$\theta_0 = \left[a_{n-1} \quad a_{n-2} \quad \cdots \quad a_0 \quad b_m \quad b_{m-1} \quad \cdots \quad b_0 \right], \quad (6)$$

where $u(\cdot)$ and $y(\cdot)$ denote the input and the output signals, $[b_m, b_{m-1}, \dots, b_1, b_0]$ and $[a_{n-1}, \dots, a_1, a_0]$ are the coefficients of numerator and denominator polynomials. Scalars m and n represent the degrees of the numerator and the denominator, respectively. Hence, the pulse transfer function can be expressed as

$$H(z) = \frac{b_m z^m + b_{m-1} z^{m-1} + \cdots + b_0}{z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \cdots + a_0}. \quad (7)$$

The dynamic system to be identified was augmented, similarly to Eq. (3), by splitting it into two parts. The first part represented the transfer function from the measurable excitation to the output, and the second part represented the transfer function from the reference signal to the same output. The estimated output of the system was then the sum output of two subsystems

$$\hat{y}(k) = \varphi_p^T \hat{\theta}_p + \varphi_d^T \hat{\theta}_d, \quad (8)$$

where $\varphi_p(k)$ contains the regression variables sampled at the plant (i.e. the system to be identified), and $\hat{\theta}_p$ contains the parameters of the plant. Respectively, $\varphi_d(k)$ and $\hat{\theta}_d$ are the regression variables and the estimates of the dynamic subsystem between the disturbance and the output. Standard least-squares identification gives estimates for the coefficients

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T Y, \quad (9)$$

where

$$\hat{\theta} = \begin{bmatrix} \hat{\theta}_p \\ \hat{\theta}_d \end{bmatrix}; \quad \Phi = \begin{bmatrix} \varphi_p^T(1) & \varphi_d^T(1) \\ \vdots & \vdots \\ \varphi_p^T(k) & \varphi_d^T(k) \end{bmatrix}; \quad Y = \begin{bmatrix} y(1) \\ \vdots \\ y(k) \end{bmatrix}. \quad (10)$$

2.2. Active Control

The active control system used consisted of a collocated feedback controller and a non-collocated feedforward controller (Fig. 4). Collocation means that the sensor and the ac-

tuator are placed at the same location. Collocation offers the advantages of providing certain stable controller implementations regardless of the system dynamics (for an idealised case).¹⁴ In the rotor test rig, the actuator and the displacement sensors at the endpoint of the rotor were assumed to be sufficiently close to each other. Thus, the feedback controller used signals measured at the rotor endpoint as its control error signals whereas the feedforward controller used the signal measured at the midpoint. The outputs of both controllers were summed and applied to the same electromagnetic actuator located at the end of the rotor. For both control topologies concerned, similar controllers with the same parameters were used in both orthogonal radial directions (except that the systems models in the feedforward controllers were slightly different).

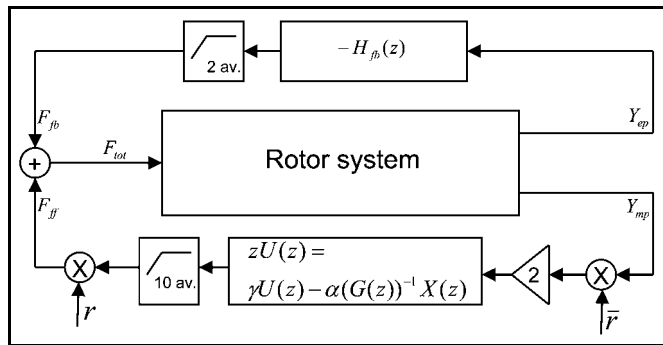


Figure 4. The control systems: feedback above, feedforward below. The reference signal r and its conjugate were used to compute the Fourier coefficients and to return to the time domain.

Two proportional-derivative (PD) controllers together with an averaging low-pass filter were applied in the feedback system. The transfer function, represented as the forward difference approximation, from the displacement at the rotor endpoint, $Y_{ep}(z)$, to the feedback force, $F_{fb}(z)$, at the actuator was

$$H_{fb}(z) = \frac{F_{fb}(z)}{Y_{ep}(z)} = -\left(\frac{K_D}{h} \frac{(z-1)}{z} + K_P\right) \left(\frac{z+1}{2z}\right), \quad (11)$$

where K_D is the derivative gain, K_P is the proportional gain, h is the sample time, and z is the discrete Z -domain variable. The PD controller was predominantly derivative in order to increase the damping of the system. This feature is discussed and justified in the next section by explaining the significance of sufficient damping from the feedforward control point of view. The proportional term was minimal, because a load-carrying effect was avoided. We wanted the actuator to have a minimal effect on the stiffness of the rotor. Also, the proportional term was not utilised significantly, because its purpose was not to increase the stability margins, whereas the derivative term was used to improve the stability. Note again that for the ideal collocated systems, the phase lead compensator (or velocity feedback) is always stable.¹⁴ This feature results from the fact that poles and zeros alternate for collocated systems. The requirement of an ideal system refers to a phase-lead compensator that does not suffer from phase lag, owing to sampling delay at a high frequency.

The principle of the Convergent Control algorithm was presented by Knospe.¹⁵ A similar control system, called Higher Harmonic Control, was presented by Hall and Wereley,¹⁶ and

applied by Sievers and von Flotow.¹⁷ All the applications were developed in order to compensate harmonic responses of rotating objects. The feedforward controllers applied in this study follow the above mentioned references. The response of a rotor (at one frequency) can be expressed by using Fourier coefficients of the excitation and of the system frequency response

$$Z(k) = TU(k) + Z_0(k), \quad (12)$$

where the $Z(k)$ represents the vibration in the output of the system, T is the frequency response of the system, $U(k)$ is the system input (open-loop control), and is $Z_0(k)$ the synchronous excitation due to rotation. The index k again refers to the discrete control steps. Minimisation of the quadratic control error function yields, for the integrative adaptation law

$$U(k+1) = U(k) + AZ(k), \quad (13)$$

where A is the (pseudo)inverse of the system response.¹⁵ In order to adjust the convergence and the stability properties, the integrative adaptation law was armed with two coefficients

$$U(k+1) = \gamma U(k) - \alpha(G(i\omega))^{-1}Z(k), \quad (14)$$

where α is the convergence coefficient (less than unity) and $G(i\omega)$ is the model of the system computed at the complex frequency $i\omega$, the compensation frequency. The coefficient γ is the integrator's leak coefficient; it corresponds to the leaky feature used in adaptive finite impulse response filter (FIR) algorithms. The leaky coefficient was given a value of unity during sub-critical operation. For supercritical speeds, it was changed in order to restrict the control force; the coefficient was then slightly less than unity. The control force was limited in order to limit the amplitude at the non-drive end of the rotor. The amplitude had to be limited due to the restricted air gap between the actuator and the rotor armature.

The control law discussed above was represented in terms of Fourier coefficients, i.e. in the frequency plane. They were used to compute the correlation between the response and the reference signal. The Fourier coefficients were computed at each time instant and the results were averaged over ten successive samples. They were computed by multiplying the measured output error signal by the complex reference signal at the frequency of rotation (Fig. 4).^{17,18} Next, the feedforward control output was computed in terms of Fourier coefficients, as defined in Eq. (14). The feedforward compensation signal is converted into the time domain by multiplying with the reference signal and taking the real part of this product

$$F_{ff}(k) = \text{Re}(U(k)r(k)). \quad (15)$$

Two systems were implemented in total, one in each orthogonal radial direction. Note that the Fourier coefficients can be interpreted as one spectral line if averaged over the period of a signal. In this study, they were not averaged over a complete period. For this reason, it would be more appropriate to refer to them as an averaged correlation signal.

The model inverses for the feedforward compensation were represented as linear polynomial functions of the rotation speed. The real and the imaginary parts of both transfer func-

tion inverses were described separately using fourth order functions. This was considered to be a practical approach since the functions were convenient to fit beforehand and the use of the fitted functions reduced the computational effort in the control unit.

2.3. Why Use Feedback Control?

As seen in the forthcoming results and also in previous studies by the author,³ the feedforward control system was mainly responsible for the reductions in the responses. This was because the major excitations occurred at the frequency of rotation. The question about the relevance of any feedback control system may be raised. However, the existence of the feedback system is justified, because it provides a certain amount of damping to the system. This helps feedforward control to work by providing smooth phase characteristics. These claims are argued below with an example.

For adaptive feedforward compensation systems, the model used by an adaptation algorithm has to describe the phase of the plant within $\pm 90^\circ$ (the original work by Ren and Kumar, 1989).¹⁹

Note that the derivation of the requirement contained assumptions of small gains and slowly changing dynamics of the plant. Note also that the analysis gave the requirements for the modelling accuracy of the system gain, but this requirement is usually less stringent if the convergence coefficient α is chosen conservatively. The accuracy requirement of $\pm 90^\circ$ is often considered loose and relatively easy to meet. However, according to our experience, the requirement can be difficult to meet in the vicinity of lightly damped poles or zeros where the phase varies rapidly. Especially, for lightly damped mechanical systems, the limit can be exceeded if the system parameters are estimated incorrectly.

Consider the complex response of a single resonance of the plant to be compensated

$$H_p(i\omega) = \frac{1}{\omega_p^2 - \omega^2 + i2\zeta_p\omega_p\omega}, \quad (16)$$

where ω is the angular frequency variable, ω_p is the natural angular frequency, and ζ_p is the damping of the plant. We assume that this resonance dominates the phase characteristics in the frequency band of interest. Also, we assume the order of the system in Eq. (16) has been estimated correctly. Thus, the model of the plant can be expressed as

$$H_m(i\omega) = \frac{1}{\omega_m^2 - \omega^2 + i2\zeta_m\omega_m\omega}, \quad (17)$$

where ω_m and ζ_m are the estimated natural frequency and the damping of the model used by the feedforward compensation system, respectively. Let us assume that this resonance determines the phase characteristics in the frequency band of interest, say from ω_1 to ω_2 . Consider then the phase errors caused by an incorrect estimate of the natural frequency, or the relative damping.

$$E_{phase} = \max(|\angle H_m(i\omega) - \angle H_p(i\omega)|), \quad \omega \in [\omega_1, \omega_2]. \quad (18)$$

Figure 5 shows the phase error contour curves between the model and the plant as a function of the modelling error

and the relative damping. The contour plot shows the phase error curves of 60° and 90° ; the former representing a limit for an appropriate performance and the latter the absolute stability limit.

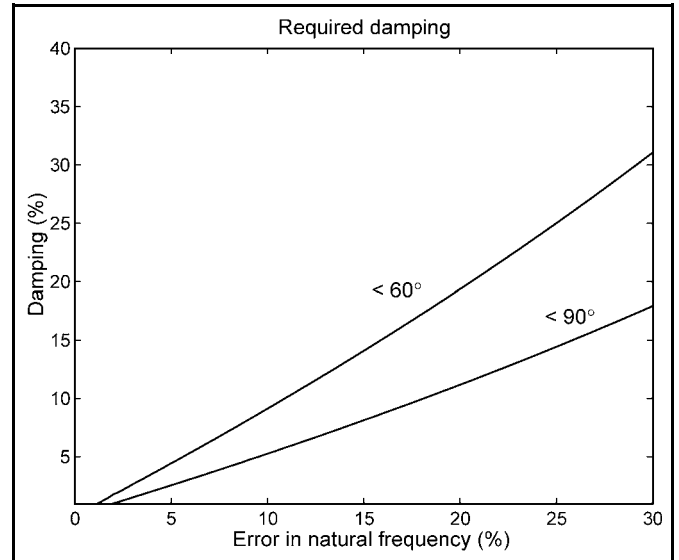


Figure 5. The phase error contour curves as a function of the modelling error in the natural frequency and the relative damping. The largest stability margins are close to the Y-axis.

In particular, relatively small modelling errors in the natural frequency can cause large phase errors for a lightly damped system because of the rapid phase changes in the resonance region. For instance, if the natural frequency estimate had an error of 5%, and the damping was less than 3%, it could lead to an unstable feedforward system. In Fig. 5, the damping was assumed to be estimated correctly. Figure 6 shows the phase error contours as functions of errors in natural frequency and damping; 10% of nominal damping was assumed. The plot implies that the phase errors were not as sensitive to the errors in the estimated damping as they were to the errors in the natural frequency. Again, this was due to rapid phase change in the resonance region.

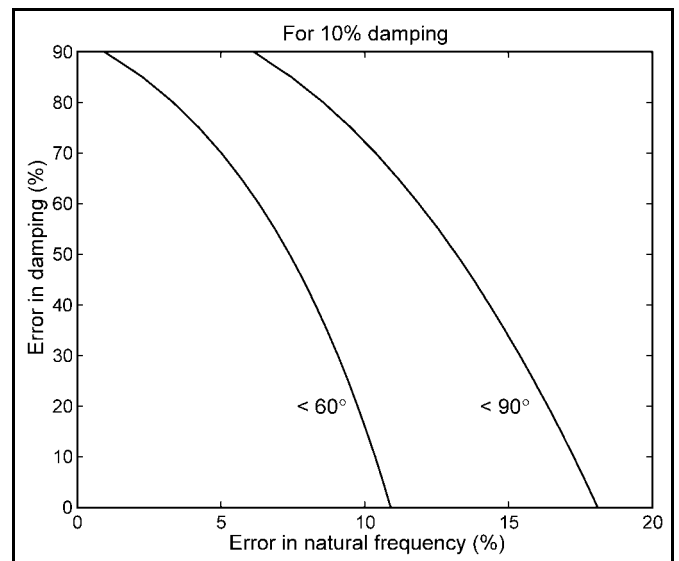


Figure 6. The phase error contour curves as a function of modelling error in the natural frequency and in the relative damping.

3. RESULTS

3.1. Identification of the Test Rig

The identification method presented was applied to the rotor test rig. The rotor was excited with band-limited white noise separately in each direction. The white noise in the band 0 to 200 Hz was fed into the actuator, and the responses were recorded at the sensor locations at different speeds of rotation. The reference signal was generated from the rotation speed sensor signal. The reference signal was a sinusoidal signal containing only the frequency of rotation. Thus, the disturbances, or forced vibrations, at the frequency of rotation only were compensated in the identification. The identification was then performed off-line, using the least squares fit according to Eq. (9). Exciting the rotor properly was difficult because of a sharp resonance and a relatively small air gap between the actuator and the rotor armature. A low excitation level was required in order to prevent the rotor from hitting the actuator. On the other hand, the excitation level was not high enough to excite higher frequencies above the first resonance of the rotor. For these reasons, the identification was carried out with the active feedback control system on. A collocated proportional-derivative control with a low-pass filter was used, as in Eq. (11). The proportional gain was equal to 7 N/mm. The derivative gain was equal to 43 Ns/m.

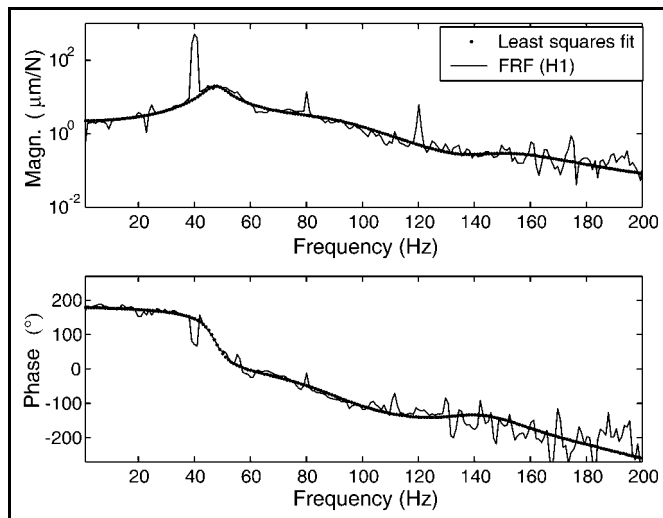


Figure 7. The frequency response function from the actuation point to the rotor midpoint when running at 40 Hz. The proposed method distinguished the rotor resonance from the forced vibration.

In identification, the order of the dynamic model to be fitted was chosen $m = 5$ (numerator) and $n = 6$ (denominator). The orders were selected to correspond to the estimated number of significant natural modes of the system. It was assumed that the three disks determine the three most significant natural modes. The disturbance model was a two-tap FIR filter as a minimal filter for a sinusoid at one frequency. The identification was carried out at different speeds of rotation. Figures 7 and 8 show the least squares fits with the disturbance compensation method presented in comparison to the direct frequency response function measurement (FRF) without any disturbance compensation. The disturbance compensation was able to remove the peak due to forced vibration at the frequency of rotation and exclude the peak from

the actual dynamics. The identification results showed that the first critical speed was dominating at about 50 Hz. The rotor had another resonance at about 130 Hz, but it was weakly observable at the midpoint.

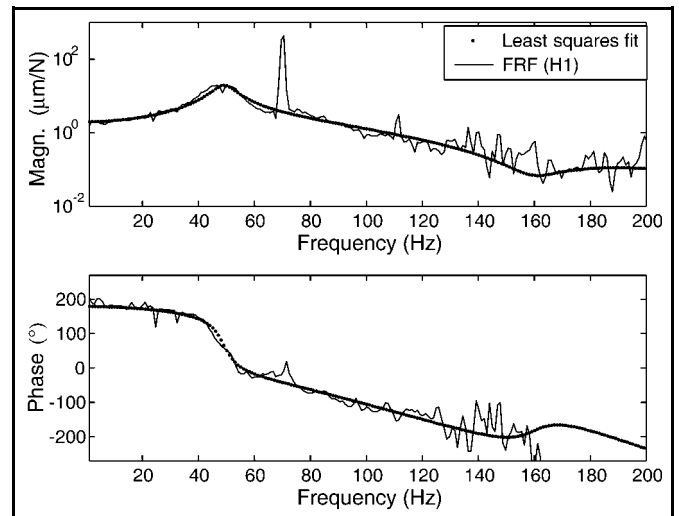


Figure 8. The frequency response function from the actuation point to the rotor midpoint when running at 70 Hz.

3.2. Attenuation with Active Control

The tests were run at different speeds of rotation in three modes: without active control, with feedback control working alone, and with feedback and feedforward control working together. Figure 9 shows the responses at six different speeds of rotation. The responses presented are the RMS values of the radial displacements computed in the frequency band from 0 to 1 kHz.

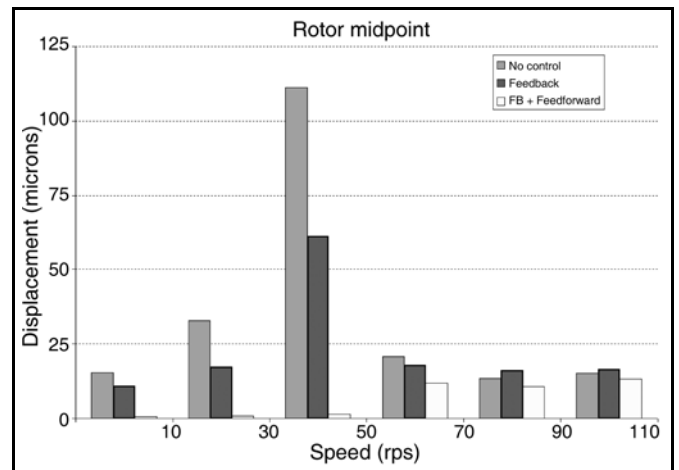


Figure 9. The responses at the midpoint without active control, with the feedback controller, and both controllers working.

The responses and the relative changes and the control forces used are presented numerically in Table 1. The following parameter values were used in the experiments: $K_D = 86$ Ns/m (derivative gain), $K_P = 7$ N/mm (proportional gain), $h = 0.0001$ s (sample time), and $\alpha = 0.02$ (convergence coefficient). The leaky coefficient (γ) was less than unity at speeds of 70 rps and higher (see Table 1 for the exact figures). The leaky coefficient was used to reduce the control force at higher speeds of rotation in order to prevent the rotor arma-

ture from hitting the actuator. Unfortunately, this action reduced the effectiveness of the control at higher speeds. The response magnitude at 50 rps without any active control was estimated because it was not possible to run at that speed without the rotor hitting the actuator.

Table 1. Tests results at speeds from 10 to 110 rps.

Rotor speed (rps)	Control method	Leaky in FF control γ	Radial displacement		Relative drop in displacement		Control force actuator (N)
			midpoint (μm)	endpoint (μm)	midpoint (%)	endpoint (%)	
10	I	1	15	107			0
	II		11	91	29	15	0.8
	III		1	52	97	51	2.3
30	I	1	33	137			0
	II		17	95	48	31	1.7
	III		1	63	97	54	2.5
50	I*	1	111	194			0
	II		61	101	45	48	2.9
	III		1	126	99	35	2.6
70	I	0.99995	21	83			0
	II		18	46	14	44	1.8
	III		12	141	43	-71	4.8
90	I	0.9999	13	39			0
	II		16	29	-20	26	1.5
	III		11	121	21	-214	7.7
110	I	0.9995	15	45			0
	II		16	37	-8	18	2.3
	III		13	65	12	-46	4.6

I – no control
 II – feedback
 III – feedback and feedforward
 * – magnitude estimate

4. CONCLUSIONS

Vibration attenuation at the midpoint of the rotor with an actuator located outside the bearing span was found effective. At sub-critical speeds the remaining controlled response was from -20 to -15 dB compared with the uncontrolled response at the midpoint. For supercritical speeds, the remaining responses were larger, because of restrictions in the use of active control at supercritical speeds. First, attenuation at the midpoint was realised at the expense of amplification at the actuator (or at the rotor endpoint). This behaviour was considered understandable, because of the phase change over the critical speed. Second, a more practical restriction was the limited control authority at the actuator. The control authority was limited by the leaky coefficient, less than unity, in order to keep the rotor armature from hitting the actuator. A limited amplitude at the actuator caused limited achievable damping at the midpoint when running above the critical speed.

The identification scheme presented, called here the Disturbance Tolerant Identification, made it possible to automatically ignore the forced vibrations due to rotor mass imbalance. The key ideas were to augment the system to be identified so as to have separate dynamics for the measurable excitation and non-measurable excitation (i.e. the load disturbance). A reference signal, correlated with the non-measurable excitation, was then used as the input replacing the non-

measurable excitation. Two dynamic systems were then identified: one from measurable excitation to the output representing the actual plant, and another from the reference signal to the output representing transfer of the forced vibrations. In the present experiments, the reference signal contained only one frequency, the frequency of rotation. The reference signal also contained other frequencies if several harmonic components were to be removed. This approach was tested and worked well, making the identification procedure more automatic. The operator did not have to manually remove the peaks due to forced vibrations because they were identified as a separate system and were not mixed with the actual plant dynamics. Currently, the system worked off-line, but could be run on-line, by using recursive least squares algorithms, for instance. An FIR filter was found better solution to filter the reference signal than an IIR filter. The FIR filter only changed the phase and the gain of the reference signal which was sufficient for the purpose.

The discussion about the model accuracy and the importance of sufficient damping in feedforward compensation led to engineering design considerations. It was shown that the requirements on modelling accuracy could be tight if an adaptive feedforward controller system was applied alone to control a plant with low damping. The plots shown helped in estimating the required model accuracy and the need to use other damping methods together with a feedforward system. The present study used a simple non-optimal dissipative feedback controller to add damping to the system. The most important role of this feedback system was to increase the total damping and make the phase behaviour smoother. This role of the feedback system was more important than the contribution to the attenuation of vibrations. Thus, the feedback control system provided feasible conditions for the feedforward system. The discussion also indicated that it may be safe to exaggerate the damping in the model. A smoother phase curve would cause smaller maximum errors than an absolutely correct phase curve, if a modelling error in the natural frequency existed. The synthesis of the feedback controller was straightforward by virtue of the collocated derivative controller. This was due to the commonly known fact that collocated velocity-feedback systems are stable. Note that the statement is true only for ideal phase-lead controllers with no phase lag at high frequencies.

In the future, the aim is to test the methods described in a larger test environment. The magnitude of the forces required indicated that resonance vibration control of larger rotors should be completely feasible. Also, some new control methods with adaptive and learning features may be tested to make the tuning of the controller more automatic. The scale of the test set-up was relatively small and the working environment was favourable for actuation (rigid foundation, no other disturbances). These factors will present new challenges to the test rig of the future.

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