# A Review on Vibration Damping in Sandwich Composite Structures

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In applications where the use of lightweight structures is important, the introduction of a viscoelastic core layer, which has high inherent damping between two face sheets, can produce a sandwich structure with high damping. Composite sandwich structures have several advantages, such as their high strength-to-weight ratio, excellent thermal insulation, and good performance as water and vapour barriers. So in recent years, such structures have become used increasingly in transportation vehicles and other applications. Care must be taken in their design to ensure that their sound isolation capabilities are adequate because coincidence generally occurs at a lower frequency for sandwich panels than for typical metal panels. Passive damping properties of composite sandwich panels are important because the damping properties affect their sound transmission loss, especially in the critical frequency range, and also their vibration response to excitation. Research on damping in sandwich composite structures is reviewed in this paper. This review includes analytical approaches, finite element models, statistical energy analysis, and damping measurement techniques. Other mechanical properties of composite sandwich structures, for example, stiffness and damage tolerance, affect each other and in turn are affected by damping. The overall effects of damping and other factors on structural response and sound radiation of composite sandwich structures are reviewed.

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## 1. INTRODUCTION

A sandwich structure consists of three elements: the face sheets, the core, and the adhesive interface layers. The great advantage of sandwich structures is that optimal designs can be obtained for different applications by choosing different materials and geometric configurations of the face sheets and cores. By inserting a lightweight core between the two face sheets, the bending stiffness and strength are substantially increased compared to a single layer homogenous structure, without the addition of much weight. The viscoelastic core has a high inherent damping capacity. When the beam or plate undergoes flexural vibration, the damped core is constrained to shear. This shearing causes the flexural motion to be damped and the vibrational energy to be dissipated. Additionally, the normal-to-shear coupling between the core and face sheets reduces the sound transmission. So in recent years, such structures have become used increasingly in transportation vehicles and other applications. Rao has described the applications of viscoelastic damping in automotive and aircraft structures.[1] Besides damping treatments used in structures, sandwich glass has been used in automotive side and rear windows to reduce noise. Composite materials are also used in pipes and ducts.[110] Nakra has published a series of reviews on vibration control with viscoelastic materials.[2-4] Trovik has summarised the major uses of constrained layer damping treatments up to 1980.[5] A thorough review of work in fibre-reinforced composite material damping research has been given by Chandra et al.[6] Some classical books and book chapters give more details on viscoelastic damping and sandwich structures.[7-17]

## 2. ANALYTICAL THEORIES

When a damping layer is attached to a vibrating structure, it dissipates energy by direct and shear strains. When a solid beam or plate is bending, the direct strain increases linearly with distance from the neutral axis. So, unconstrained damping layers, which dissipate energy mainly by direct strain, are attached to the remote surfaces. On the other hand, the shear stress is the largest at the neutral axis and zero on the free surfaces. Therefore, constrained layers dissipate energy by the action of shear stresses. It has been shown that shear damping in viscoelastic materials is higher than in typical structural materials. The constrained treatment has higher stiffness than the unconstrained damping treatment. For these reasons, sandwich composite structures are widely used.

### 2.1. Damping in Sandwich Beams and Plates

Since the late 1950's, many papers have been published on the vibration of sandwich structures. The Ross-Ungar-Kerwin model is one of the first theories which was developed for the damping in sandwich structures.[18-21] In Kerwin's initial study, an analysis was presented for the bending wave propagation and the damping in a simply supported three-layer beam.[18] One of the limitations of this analysis is that the bending stiffness of the top layer must be much smaller than that of the bottom layer. Ungar generalised the earlier study and derived an expression for the total loss factor of sandwich beams in terms of the shear and structural parameters.[20] Based on such an expression, two important conclusions can be drawn. First, if the constraining layer is thinner than the viscoelastic damping layer, then the system damping has a maximum value when the shear parameter of the core has an optimal value in the intermediate range, as shown in Fig. 1, where $X$ and $Y$ are the shear and structural parameters and $\beta$ is the damping in the viscoelastic layer. Second, the loss factor has a maximum value when a three-layer sandwich structure is symmetric about the neutral axis.

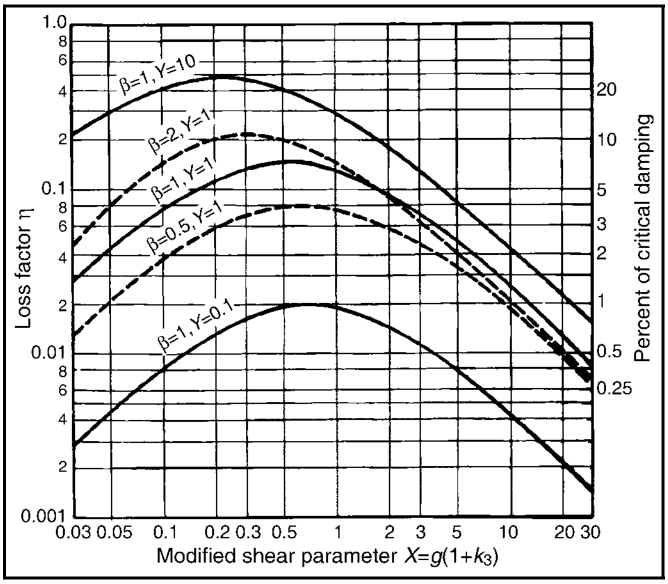


**Figure 1.** The effect of the shear modulus on the total damping in a sandwich structure.

Ruzicka summarised earlier research on viscoelastic shear damping mechanisms and presented several structural damping design configurations, especially the so-called "cell-insert" idea.[22,23] He stated that the loss factor is independent of the stress level for pure viscoelastic materials. He also analysed the dynamic properties of viscoelastic-damped structures using a lumped-parameter model which resulted in a number of conclusions that agree with those obtained from the flexural wave analysis discussed in reference 18.

The limitations of Kerwin's model have been avoided in Yu's theory by using a variational approach.[24] Yu took into account inertia effects due to transverse, longitudinal and rotary motions and then considered the combined effects of the three loss factors associated with the shear and direct stresses of the core and with the direct stress in the face sheets. However, Yu only studied the flexural vibration of symmetric sandwich plates. Sadasivia Rao and Nakra analysed the damping in unsymmetric sandwich beams and plates and also included the inertia effects of transverse, longitudinal and rotary motion.[36] Inclusion of all the inertia effects in the flexural vibration analysis gives three families of modes in bending, extension and thickness-shear.

In extending the work of Kerwin, DiTaranto derived a sixth-order linear homogeneous differential equation for freely vibrating beams having arbitrary boundary conditions.[25-27] In this model, the modes are completely uncoupled, which greatly simplifies the general forced vibration problem. However, the loss factor calculated by using this equation does not depend on the boundary conditions. This conclusion obviously cannot be correct. Mead and Markus modified the theory and studied different boundary conditions in terms of the transverse displacement.[28,29] Using the separation of variables method, they derived the natural frequencies of sandwich beams and studied the effects of the shear and structural parameters on damping. The relationship is similar to the equation derived in reference 20. Mead and Markus proved that the loss factor $\eta$ is much less sensitive to the change of the shear parameter when the structural parameter $Y$ is large, as shown in Fig. 2. They also showed that the maximum values of the damping are not very sensitive to the boundary condition, while different boundary conditions shift the frequency at which the maximum damping occurs.

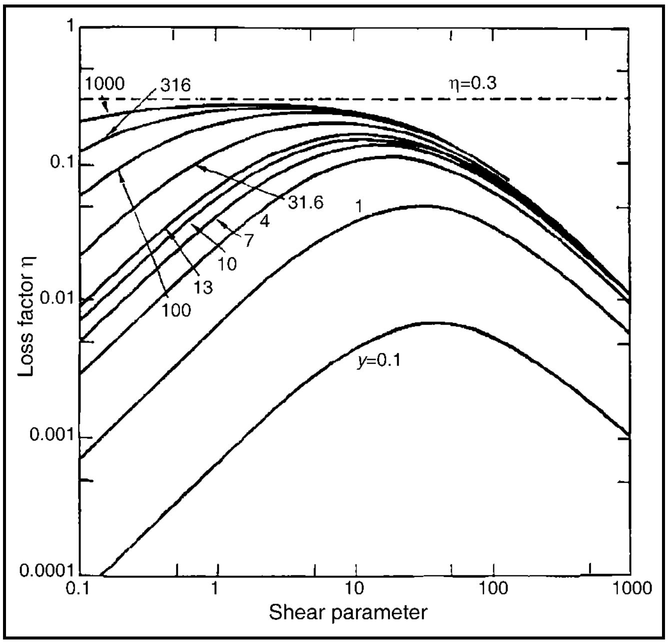


**Figure 2.** The effects of the shear and structural parameters on the system loss factor.

In another study, Yan and Dowell initially included the effects of face sheet shear deformation and of longitudinal and rotary inertia.[34] However, from the set of equations obtained, the longitudinal and rotary terms were neglected by assuming the face sheets to be very stiff in shear. This procedure results in a fourth-order partial differential equation. Mead analysed the damping in symmetric sandwich plates with one pair of opposite edges simply-supported.[30] He also studied the effect of different boundary conditions for the other edges and derived a sixth-order equation. Mead compared the difference between the fourth-order model derived by Yan and Dowell and the sixth-order model.[31,33] Based on Mead's sandwich plate model, Cupial and Niziol included the shear deformation of the face layers and rotary inertia and then studied the simply supported sandwich plates.[47] The damping calculated by using the shear deformation model is somewhat lower than that obtained from Mead's model. Wang and Chen studied damping in annular sandwich plates using a similar method.[48]

Since high damping is usually associated with relatively low stiffness and strength, it is a good idea to increase the stiffness using multi-span sandwich structures. Mead extended his previous work to periodically supported sandwich plates.[32] The basic idea is that at a particular frequency, all the displacement and forces at a point in one periodic element are identical to those at the corresponding point in the adjacent element, apart from a phase difference which is determined using an iterative technique. The dependence on frequency and the effects of support spacing and shear parameters on damping were also studied. Rao and He also analysed damping in multi-span sandwich beams.[46] Rao and He derived two sixth-order differential equations to govern the transverse and longitudinal motions for each span using Hamilton's principle. The effects of the thickness of the face sheets and core and of the location of the intermediate support on the damping were studied for a two-span sandwich beam.

Rao derived a similar equation of motion using Hamilton's principle.[38] He presented an extensive study using computer programs to predict the loss factor and natural frequencies for different boundary conditions in terms of the shear parameter. Rao also analysed the flexural vibration of short unsymmetric sandwich beams including all the higher order effects, such as rotary inertia, bending, extensional and shear effects, in all the layers.[37] He compared the loss factor and natural frequencies obtained by using this new model and earlier models. For a beam where the core is thicker than the face sheets, all the models predict identical results, although Rao's model includes the higher order effects. This means that for thick core beams, the effects of rotary inertia, extension and shear are insignificant in all the layers.

All the researchers introduced above, except Yu, have only considered the contribution of the damping in the viscoelastic core to the total damping in the entire structure by using the complex form of the shear modulus for the core. An advantage of using the complex shear modulus is that the differential equations only contain the even order terms. So they are easy to solve. These models are all based on the following assumptions: (a) the viscoelastic layer undergoes only shear deformation and, hence, the extensional energy of the core is neglected; (b) the face sheets are elastic and isotropic, and the shear energy contributed in them is neglected; and (c) in the facings, the plane sections remain plane and normal to the deformed centrelines of the facings. Mead conducted a comprehensive study on a comparison of these models in reference 33.

Instead of only considering the damping in the core, Ungar and Kerwin also proposed the so-called modal strain energy (MSE) model in order to include the damping capacities of all the elements. In this model, the damping of the material can be characterised by the ratio of the energy dissipated in each element to the energy stored in the material.[21] Based on the MSE method, Johnson and Kienholz produced a method to predict damping in structures with constrained viscoelastic layers by using finite element analysis.[75] Hwang and Gibson studied damping in composite materials and structures at both macromechanical and micromechanical levels using the MSE method.[39-42] They studied the contribution of interlaminar stresses to damping as well.[43]

The frequency dependence property of viscoelastic damping was first presented by Lazan.[49] Ruzicka and Mead came to similar conclusions using lumped-parameter models.[16,22] Mead also studied the influence of the boundary conditions on the frequency dependence of the loss factor.[29] Nilsson used Hamilton's principle to derive two sixth-order differential equations which govern the bending of sandwich beams.[50,51] The behaviour of a sandwich structure in the low frequency region is determined by pure bending of the entire structure. In the middle frequency region, the rotation and shear deformation of the core become important. At high frequencies, the bending of the face sheets is dominant. Therefore, if the damping in the core is higher than that in the face sheets, then the overall damping has a maximum value in the middle frequency range. On the other hand, if the damping in the core is less than that in the face sheets, then the total damping has a minimum value in the middle frequency range. Figure 3 shows the calculated total loss factors for three different cases, where the loss factor in the core $\eta_2$ is set to be 2% and the loss factor in the face sheets $\eta_1$ varies.

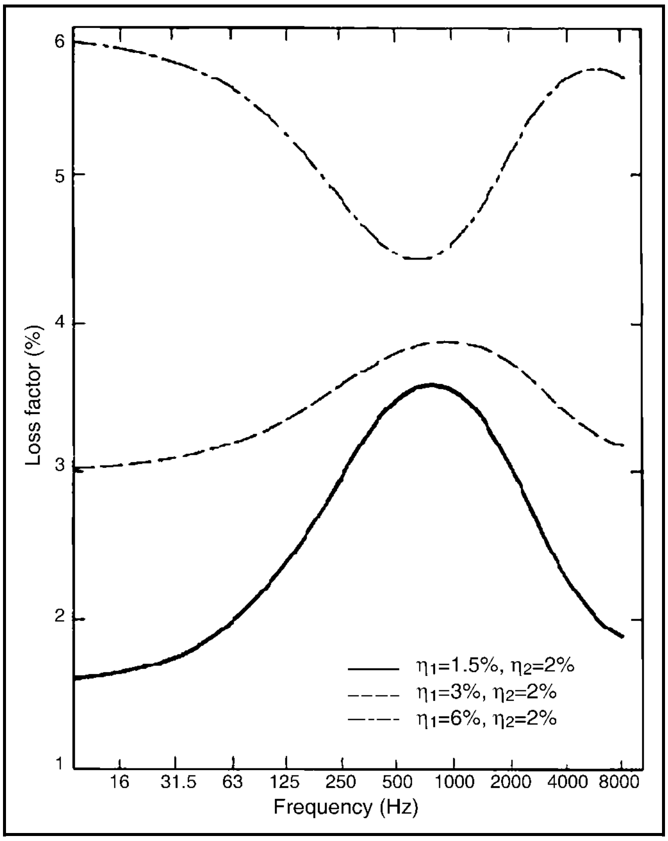


**Figure 3.** The frequency dependence of the damping in sandwich structures.

A. Nilsson and C. Nilsson also studied the dynamic properties of sandwich structures.[51] Based on the sixth-order differential equations, the wavenumber of a sandwich beam can be solved by assuming simple harmonic solutions. In addition, the apparent bending stiffness, $D$, can be calculated from $D = \omega^2 m / k_b^4$, where $\omega$ is the angular frequency, $m$ is the mass per unit length, and $k_b$ is the bending wavenumber. Since the motion of a sandwich beam is dominated by the face sheets, which are limp in the high frequency range, the apparent bending stiffness approximates to the bending stiffness of the face sheets when the frequency increases. Figure 4 demonstrates the measured and predicted apparent bending stiffness of two beams cut from the same Nomex honeycomb sandwich panel in two perpendicular directions. Note that the bending stiffness presented by Nilsson is for beams with unit width, so the unit is Nm, rather than $Nm^2$.

Nilsson also presented the sound reduction index (sound transmission loss) of sandwich panels.[50] Figure 5 shows the measured and predicted sound reduction indices of a sandwich panel and face sheet.

In Nilsson's research, the cores are either honeycomb or solid viscoelastic materials.[50] Li and Crocker studied the frequency dependence of damping in sandwich beams with combined honeycomb-foam cores.[52] Because of the viscoelastic property of the foam, the damping in the core is greater than that in the face sheets. The honeycomb material is expected to enhance the stiffness of the entire structure. So the normal-to-shear coupling is still effective in the high frequency range and, thus, the damping is increased substantially. The effects of thickness and delamination on damping have also been analysed. Figure 6 illustrates the frequency dependence of symmetric sandwich beams with different configurations, where the thickness of a single-layer face sheet is 0.33 mm. If the face sheet thickness increases, the damping in the low and high frequency ranges is lower, but it is still high in the middle frequency range. If the thickness of the core increases, the damping is increased in both the middle and high frequency ranges.

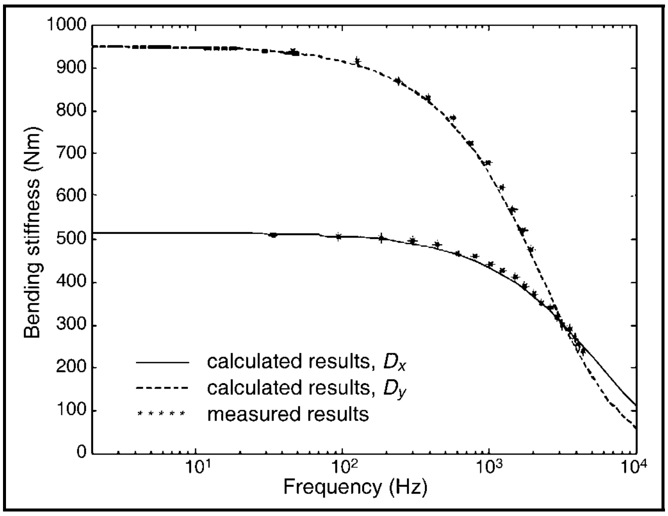


**Figure 4.** Bending stiffness for two beams cut from the same honeycomb sandwich panel.

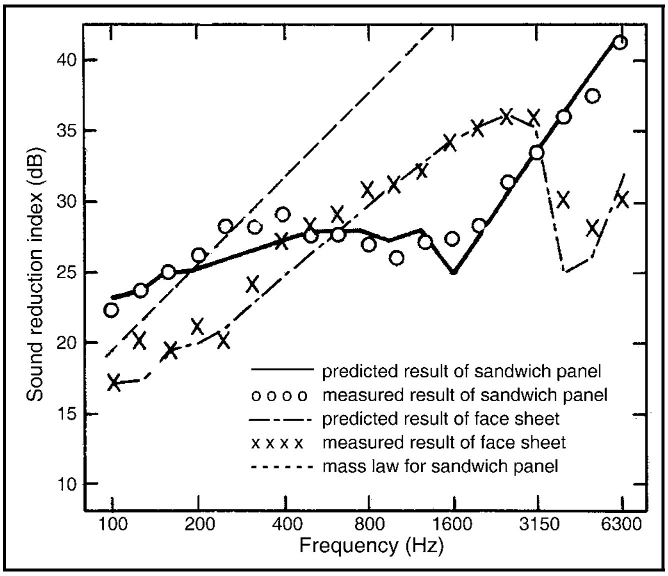


**Figure 5.** Measured and predicted sound reduction indices.

The theoretical models discussed so far can be categorised into two classes, fourth-order models and sixth-order models. Models derived by Mindlin's theory and Timoshenko's theory both lead to a fourth-order differential equation. Mead[31,33], Rao[37], and Nilsson[51] all show that sixth-order models lead to more accurate results for the dynamics and damping of sandwich panels than fourth-order models. Nilsson states that due to the frequency dependence of sandwich structure properties, solutions of the fourth-order differential equation agree well with measurements obtained at low frequency. However, as the frequency increases, the calculated results disagree strongly with the measurements.

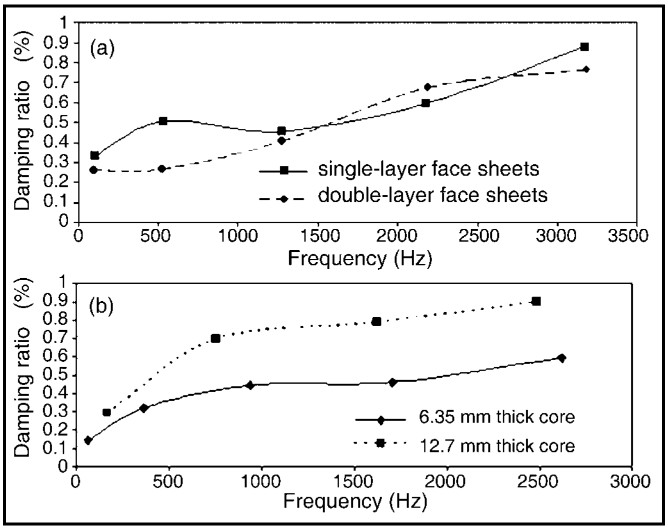


**Figure 6.** The effects of thickness on the total damping in sandwich beams with a foam-filled honeycomb core.

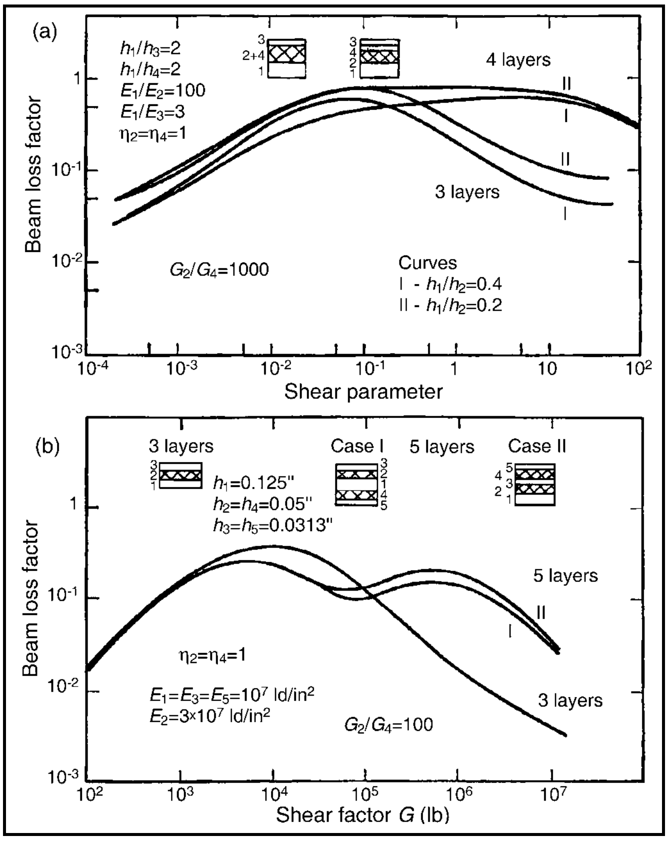


**Figure 7.** The shear parameter effect on the total damping in multi-layer sandwich beams.

Besides the three-layer sandwich structures, multi-layer sandwich structures are also widely studied.[53-60] Grootenhuis showed that the four-layer and five-layer beams have a wider high damping range in terms of the damping layer shear modulus than the three-layer sandwich beams, as shown in Fig. 7, where $E$ and $G$ denote the Young's modulus and the shear modulus, respectively, and $h$ is the thickness.[55] Asnani and Nakra studied the damping characteristics of symmetric multi-layer beams with identical viscoelastic and elastic layers alternatively arranged.[57] They provided three design criteria and analysed the effects of the shear parameter and layer thicknesses on the total damping. Alam and Asnani extended the previous work to multi-layer structures with orthotropic damping layers where each damping layer is constrained between two elastic layers.[58-60] They considered shear strain in all the layers, but their result does not satisfy continuity of the shear stress across the interfaces. Bhimaraddi proposed a refined shear deformation theory in which the shear stresses are continuous across the interfaces.[61] Rao and He studied several different multi-layer configurations using numerical analysis.[45] Two more fibre-reinforced layers are added on the two free surfaces. The total damping can be improved by changing the fibre orientation.

Among multi-layer sandwich structures, special attention has been given to spaced sandwich structures. A spacer is inserted between the base plate and the viscoelastic damping layer to magnify the shear strain and to enhance the damping. Since the viscoelastic damping layer is separated from the neutral axis of the entire structure due to the spacer, the direct stress is increased. To make this configuration effective, the shear stiffness of the spacer must be much greater than that of the damping layer so that the shear stress in the damping layer also increases. Ross, Ungar and Kerwin presented this idea first in reference 19, as shown in Fig. 8. Nakra and Grootenhuis derived the equations of motion using Hamilton's principle.[56] The Two face sheets are assumed to be perfectly elastic, and the damping layer and spacer are viscoelastic. Compared with three layer sandwich beams and plates, multi-layer structures have a wider high damping range in terms of the core shear modulus.[55,56] Van Vuure et al. applied the modal strain energy method to model such structures and the finite element method to calculate the loss factor in each layer.[53] They also studied the effects of the spacer position.

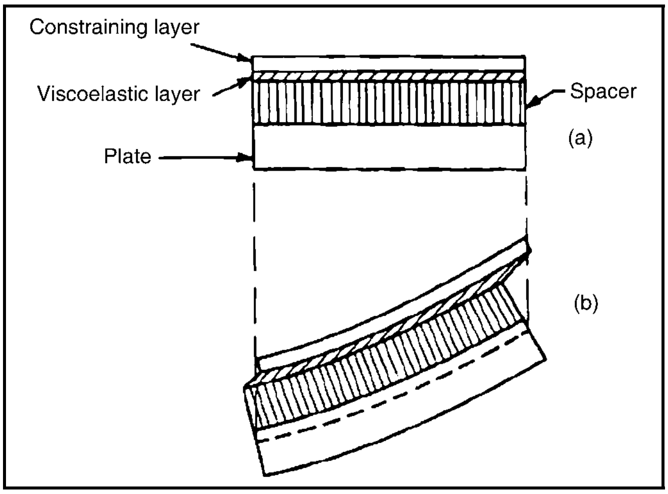


**Figure 8.** A sandwich beam with a spacer beneath the viscoelastic damping layer. (a) Undeformed beam; (b) deformed beam.

Since many complex structures are jointed, joint damping is also an interesting phenomenon. Joint fasteners for sandwich composite structures can be bolts, rivets, or adhesive layers. He and Rao analysed the damping in adhesively bonded double-strap joints.[44] The effects of the shear modulus of the damping layer and structural parameters on the modal loss factor, such as the damping and constraining layer thicknesses, are studied. Figure 9 shows that if the viscoelastic damping layer is much softer than the constraining layer, the total loss factor varies little with the shear modulus of the damping layer. In Fig. 9, the normalised shear modulus is defined as the ratio of the core shear modulus to the face sheet Young's modulus.

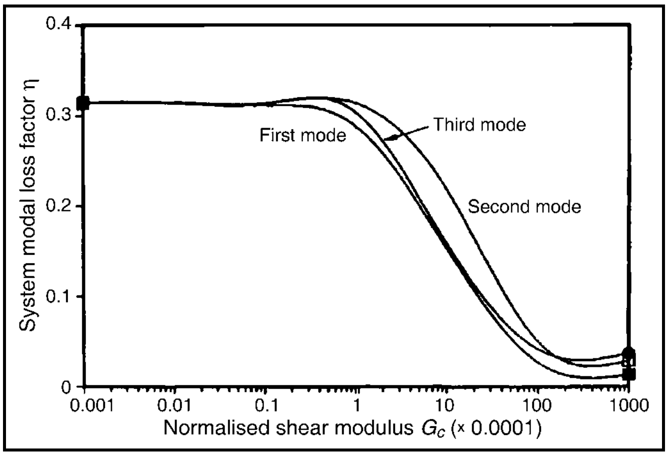


**Figure 9.** Variation of the modal loss factor with the normalised shear modulus.

In general, the damping of bonded structures tends to be lower than that of structures with bolted and riveted joints.[16] Nanda and Behera conducted a theoretical analysis and experiments for the damping in bolted laminated structures.[62] The damping in such structures depends on many factors, such as the diameter of the bolts, the tightening torque on the bolts, the number of layers, and so on.

Marsh and Hale presented a different damping configuration, which consists of an internal shear damping treatment.[63] Such structures are hollow with viscoelastic damping materials bonded inside the structures. This is very similar to the "cell-insert" concept presented by Ruzicka.[22,23] Marsh and Hale analysed the effects of geometry and mechanical parameters on the damping in such structures. Figure 10 illustrates the internal damping treatment idea.

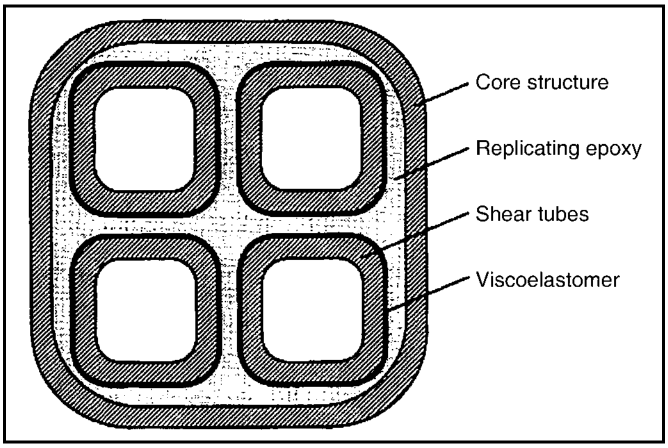


**Figure 10.** Internal damping treatment.

## 2.2. Damping and Damage

Damage is another mechanism that causes increased damping. Prasad and Carlsson analysed debonding and crack growth in foam core sandwich beams using the finite element method.[64] Crack effects on the dynamic characteristics of conventional and composite beams have been studied by many authors, including those in this issue of the journal.[111]

Experiments were carried out with cantilever beams and shear specimens.[65] Luo and Hanagus studied the dynamics of delaminated beams by using a piecewise-linear spring model to simulate the behaviour of delaminated layers.[66] Delamination introduces friction in the unbounded region of the interface, and the damping increases with the size of the delamination. Meanwhile, increased damping leads to lower natural frequencies. This effect is significant in the high frequency range.[67] Experimental results presented by Li and Crocker are found to be consistent with this prediction.[52] Figure 11 compares the experimental results that were obtained from two double cantilever beams with and without delamination. The delamination is 5% of the total length on one side of the beam.

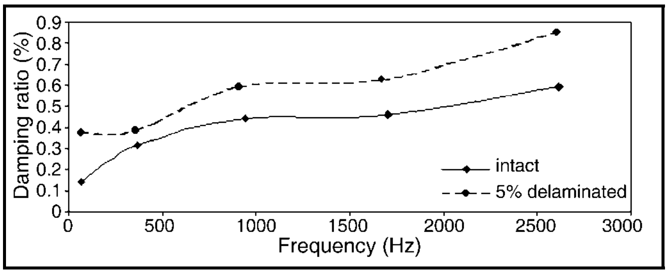


**Figure 11.** Effect of delamination on damping in a sandwich beam.

Delamination affects the stiffness of sandwich beams as well. For sandwich beams with delamination, the bending stiffness is reduced substantially. If there is delamination on both sides of the beam, the bending stiffness is reduced more than if there is delamination on only one side. This conclusion is the same as that resulting from Frostig's model which is based on high-order elastic theory.[68]

It is worth noting that high damping is not the only beneficial property for good noise and vibration control. The additional effects of many other factors, such as mass, stiffness, damage tolerance, and so on, have to be considered as well. High damping is usually associated with a relatively low stiffness. So the trade-off between the requirement for low vibration levels and strength and stiffness must be analysed during the design stage. Some criteria for assessing the damping effectiveness can be found in reference 69.

## 3. FINITE ELEMENT MODELS

The complex eigenvalue and the direct frequency response methods are two kinds of conventional approaches that can be used to evaluate damping. Lu et al. conducted a series of research studies on the vibration of damped sandwich structures using the direct frequency response method.[70-74] However, these two conventional methods are both computationally expensive. In recent years the modal strain energy method and the Golla-Hughes-McTavish (GHM) method have come into more common use.

As discussed in the first section, the modal strain energy method was proposed by Ungar and Kerwin and was realised by Johnson by using finite element analysis. Although this is an approximate technique for the prediction of damping, the advantage of this method is that only the response of undamped normal modes needs to be calculated. As a result, the energy distributions are of direct use to the designer when deciding where to locate damping layers.[75] Veley and Rao studied the effect of the thicknesses of all the layers and the amount and location of the damping treatment.[76] They claim that an increase in the constrained layer thickness increases the loss factor. Although an increase in the viscoelastic layer thickness increases the loss factor of the first mode, it decreases the loss factor of higher modes. Zambrano et al. studied the accuracy of this method for the estimation of the response of structures using viscoelastic dampers.[77] Plagianakos and Saravanos presented a new finite element model for sandwich beams involving quadratic and cubic terms for approximation of the in-plane displacement in each layer.[78] The damping is calculated using the modal strain energy method. The effects of ply orientation, thickness, and boundary conditions on the damping are analysed. Shorter used a one-dimensional finite-element mesh to describe the low order cross-sectional deformation of laminates and used the modal strain energy method to calculate the damping.[79] This finite element model showed that below a particular frequency only longitudinal, shear, and bending waves are observed. At high frequencies additional propagating waves cut on in the beam which involve the out-of-phase flexural motion of the face sheets. This is called symmetric motion or dilatational motion by other researchers.[80-82]

The GHM method is a technique for deriving a viscoelastic finite element from the commonly-used elastic finite element and for measurements of both frequency and time domain material behaviour.[83] Based on this model, Wang et al. analysed the vibration characteristics of sandwich plates incorporated with the Galerkin method and conducted experiments with simply supported and clamped plates.[84] The GHM method can successfully predict the frequency dependence of the complex shear modulus in the core.

Chen et al. presented an order-reduction-iteration approach to predict the damping in sandwich structures.[85] Such a method consists of two steps, the first-order asymptotic solution of the non-linear real eigenequation and the order-reduction-iteration of the complex eigenequation.

Nayfeh analysed five-layered sandwich beams using finite element implementation of the modal strain energy model.[86] He studied different boundary conditions and partially covered sandwich beams, the effects of the coupling factor, and the ratio between the stiffnesses of the face sheets and the core.

## 4. STATISTICAL ENERGY ANALYSIS METHOD

Finite element models are generally only efficient for problems at low and middle frequencies. Since the size of the elements must be considerably smaller than the minimum wavelength, the required number of elements increases dramatically with the frequency range of interest, as well as the geometry and complexity of the structure. The statistical energy analysis (SEA) or power balance method is attractive at high frequencies where a deterministic analysis of all resonant modes of vibration is not practical. In the SEA model, a complex structure is virtually divided into coupled subsystems. Energy flows from one subsystem to others. Based on the assumption of power balance for these subsystems, the averaged behaviour of the whole structure can be predicted. Because SEA calculates the spatial and frequency averaged response, the SEA model for a complex structure is quite simple. The modal density, the internal loss factor for each subsystem, and the coupling loss factors between the subsystems are the basic SEA parameters.

Since the SEA model is widely used in sound transmission research and damping is related to the sound transmission properties, especially at the critical frequency, the SEA is also used in damping estimations. Although this method cannot be applied for measurement of damping in an individual mode of vibration, it is very practical for the estimation of damping in a particular frequency band. Actually, this feature of SEA is experimentally very useful because the uncertainty and severe modal overlap of the frequency response functions of sandwich structures at high frequencies make it difficult to determine the loss factor for an individual mode.

Lyon has presented the concept of SEA and used this approach to formulate a model for the prediction of damping.[87] Bloss and Rao measured the damping in laminated glass for vehicle side windows using the SEA method.[88]

As mentioned before, modal density is one of the basic SEA parameters. The first theoretical study of the modal density of a sandwich shell with an isotropic core was conducted by Wilkinson based on a fourth-order equation of motion.[89] Erickson showed that for typical honeycomb structures, the effect of rotary inertia and bending stiffness of the face sheet can be neglected, but the shear flexibility of the core is important.[90] So he modified the theoretical expression for the modal density of the honeycomb plates. Clarkson and Ranky derived a new theoretical expression based on the sixth-order equation of motion.[94] This new expression gives a good estimation of the modal density of plain honeycomb plates and is independent of the shape of the structure. Renji et al. derived an expression to evaluate the modal density of a honeycomb sandwich panel with orthotropic face sheets based on a fourth-order governing differential equation.[101]

As for the experimental methods used to determine the modal density of panels and beams, Lyon and DeJong have described some basic approaches.[87] The mode count is a straightforward method, which identifies and counts the number of resonance peaks from the frequency response function. At high frequencies, severe modal overlap makes the modes indistinct. Clarkson and Pope developed an experimental technique to determine the modal density of a lightly damped structure by measuring the spatial average of the driving point mobility $Y(\mathrm{i}\omega) = V(\mathrm{i}\omega)/F(\mathrm{i}\omega)$, where $V$ and $F$ are the Fourier transforms of the velocity and force signals.[91,92] Theoretically, the real part of the driving point mobility must be positive. Several papers were published later on to improve this technique. Ranky and Clarkson demonstrated that the one-third octave bands are too wide. A more suitable bandwidth is 100 Hz for metal plates, but 500 Hz for honeycomb plates because the modal density of the sandwich structures is relatively low.[93] Because the measured velocity at the driving point includes a non-propagating (near field) component, which is not related to the energy input, the velocity of the driving point should not be included in the calculation of the spatial average velocity. Clarkson and Ranky also studied the effect of discontinuities in honeycomb plates, such as circular cut-outs, added mass, and added stiffeners.[94] In order to solve the presence of negative values in the real part of driving point mobility, Brown presented a three-channel technique by measuring one more signal $s(t)$, which is the original signal to drive the power amplifier.[95] The driving point mobility is then calculated as $Y(\mathrm{i}\omega) = G_{sv}(\mathrm{i}\omega)/G_{sf}(\mathrm{i}\omega)$, where $G$ denotes the cross-spectrum. Brown and Norton suggested a method to correct the mobility calculation error, which is introduced by the added mass between the transducer and the structure.[96] Keswick and Norton studied three different excitation arrangements for impedance head measurements and used the spectral mass method to correct the measured mobility.[98] Hakansson and Carlsson presented a similar correction method using a dual-channel FFT analyser with an unloaded impedance head.[99] Applying Brown and Norton's correction method, Renji measured the modal density of foam-filled honeycomb sandwich panels using an improved input mobility method by including both the real and imaginary parts.[102] Beyond a particular frequency, the measured modal density decreases with frequency, although the theoretical results still increase. Renji explains that this is because of the vibration of the honeycomb cells which occurs at high frequencies.

Based on the experimental techniques introduced above, the loss factors of sandwich structures are then also measured.[91,93,103] It is important to notice that the frequency average loss factor in a frequency band is not the arithmetic average of the individual modal loss factors.

If measurements are made in air, the measured loss factor is basically the total effect of the internal and acoustic radiation loss factors. Since the coincidence frequency of a sandwich structure is generally lower than thin metal plates due to both the bending and shear waves propagating in it, the radiation loss factor could be very significant in the frequency bands of interest. Clarkson and Brown, and Norton have shown that the use of the loss factor measured in air in the SEA model can lead to large errors in the estimated response because for honeycomb sandwich plates, the acoustic damping is the major component of the total damping.[97,100]

The acoustic radiation damping is effective near and above the coincidence frequency of a structure. Compared with a solid homogeneous plate of equal weight, the sandwich plate generally has a lower critical frequency due to the high stiffness-to-mass ratio. So the acoustic radiation damping in a sandwich plate is higher than that in a solid plate. However, it should be noticed that an increase in the acoustic radiation damping may be an advantage in some vibration problems, but a disadvantage in others.[16] For example, since the radiation loss factor of a sandwich structure is normally much higher than its internal loss factor, if it is excited in a diffuse sound field, then the time-averaged structural vibration levels are almost independent of the acoustic damping.[16] In addition, since the radiation loss factor is proportional to the radiation efficiency, which affects the sound transmission loss, an increase in the acoustic radiation damping leads to a reduction in the sound transmission loss.[104,105]

## 5. DAMPING MEASUREMENT TECHNIQUES

Basically, there are four measures of damping: the loss factor $\eta$, the quality factor $Q$, the damping ratio $\zeta$, and the imaginary part of the complex modulus. However, they are related to each other. The loss factor or damping ratio is used in measurements:

$$\eta = \frac{D}{2\pi W} = \frac{1}{Q} = 2\zeta = \frac{2C}{C_c} = \frac{E''}{E'} = \tan\phi \,, \qquad (1)$$

where $D$ and $W$ are the dissipated and total powers in one cycle of vibration, $C$ and $C_c$ are the damping coefficient and the critical damping, and $E'$ and $E''$ are the real and imaginary parts of the complex modulus, respectively.

Many references present reviews of damping measurements.[10,11,17,87,109] Generally, there are three sorts of experimental methods:

*Decay rate method*. This method can be used to measure the damping of a single resonance mode from a free vibration signal. The structure is given an excitation by a force in a given frequency band, then the excitation is cut off, followed by an observation of the free vibration signal. A commonly used damping calculation method for free vibration signals uses the logarithmic decrement $\delta$:

$$\delta = \frac{1}{m} \ln \frac{A_n}{A_{n+m}}, \tag{2}$$

where $A_n$ and $A_{n+m}$ are the amplitudes of the $n$-th and the $n+m$-th cycle in the free vibration signal. The damping ratio is then given by

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}. \tag{3}$$

However, the logarithmic decrement method is very sensitive to noise.

An improved approach is to obtain the envelope of the free vibration signal by constructing its analytic signal using the Hilbert transform.[106] For a given real signal $x(t)$, its analytic signal $x_a(t)$ is

$$x_a(t) = x(t) + jH\{x(t)\}\ , \tag{4}$$

where the subscript $a$ stands for analytic and the Hilbert transform of $x(t)$ is defined as

$$H\{x(t)\} = -\frac{1}{\pi} \int \frac{x(\tau)}{t-\tau} d\tau\ . \tag{5}$$

The magnitude of the vector $x_a(t)$ is the envelope of the signal $x(t)$. The damping ratio $\zeta$ can be evaluated by exponential curve fitting

$$\zeta = -\frac{\Pi_e}{2\pi f}\ , \tag{6}$$

where $\Pi_e$ is the power of the best exponential fit and $f$ is the frequency of the free vibration.

Another problem associated with the decay rate method is that this method only works for a single mode at resonance. Li and Crocker have presented a method using the Gabor analysis to decouple the modes existing in a free vibration decay signal.[106] Although the modes can be decoupled using bandpass filters, Li and Crocker proved that the reconstructed signals obtained using the Gabor analysis result in signals with higher signal-to-noise ratios than those obtained when using bandpass filters.

*Modal bandwidth method*. The half-power point method is the most common form used to determine the damping by using either impact excitation or white noise excitation and by calculating the frequency response function (FRF). The loss factor is given by the ratio of the half-power frequency band to the natural frequency:

$$\eta = \frac{f_2 - f_1}{f_n}, \tag{7}$$

where $f_n$ is a natural frequency and $f_1$ and $f_2$ are the upper and lower frequency limits of the 3 dB frequency band of this mode, respectively.

A considerable amount of attention has been devoted to the modal parameter extraction method using the FRF analysis.[107-109] This method applies only to the determination of the damping of a single mode. However, if severe modal overlap occurs in the frequency response functions, the power balance method is more practical.

*Power balance method*. The SEA method, or power balance method, has been discussed in the previous section. The damping in a particular frequency band, predicted by SEA, is based on the ratio of the dissipated energy to the total energy measured in this frequency band. Under steady state conditions, the dissipated energy is equal to the input energy. So, the loss factor in a frequency band can be determined by measuring the input power and the total energy of a modal subsystem:

$$\eta = \frac{W_{\text{in}}}{2\pi f\, W_{\text{total}}}, \tag{8}$$

where $W_{\text{in}}$ is the input power, and $W_{\text{total}}$ is the total power measured in a frequency band with the centre frequency of $f$. The input power can be calculated using the input force and modal density. The total energy of a subsystem is the product of the mass and the spatial average of the mean square value of the velocity.[87,88,103]

Damping can also be modelled using hysteresis loops. Hornig and Flowers discuss such modelling and measurements and the use of genetic algorithms in reference 112.

## 6. CONCLUSIONS

Vibration damping is a complex phenomenon. It depends on many factors, such as material internal damping, boundary conditions, temperature, frequency, level of strain/stress, and so on. Based on our literature review of damping in sandwich structures, the following conclusions can be drawn:

1) Sixth-order models lead to more accurate results on the dynamics and damping of sandwich structures than fourth-order models.

2) From a theoretical point of view, the MSE method is better than the Ross-Ungar-Kerwin model since it includes the damping in all of the elements. However, this method requires preliminary information on the damping in all the layers, which may be difficult to obtain, especially for very thin composite face sheets.

3) Finite element methods are very efficient nowadays. Some approximations, however, have to be made because some materials (for example, adhesive layers) are difficult to model.

4) The Ross-Ungar-Kerwin model indicates that the loss factor has a maximum value when a three-layer sandwich structure is symmetric about the neutral axis.

5) If the constraining layer is thinner than the damping layer, then the system damping has a maximum value when the shear modulus of the core has an optimal value in the intermediate range.

6) The significance of higher order effects varies for different configurations of sandwich structures.

7) Damping in different joints varies. For bonded joints, if the viscoelastic damping layer is much softer than the constraining layer, the total loss factor varies little with the shear modulus of the damping layer. For bolted joints, as the bolt diameter decreases, the damping increases.

8) Most multi-layer structures, such as four-layer and five-layer sandwiches, normally possess more damping than the three-layer sandwich structures. Multi-layer structures also have a wider high damping range in terms of the core shear modulus.

9) A spacer with a higher shear modulus than the damping layer can be inserted beneath the damping layer to increase the shear stress in the damping layer and, thus, to increase the energy dissipation in the whole structure.

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