Demonstrations of Acoustic Re-radiation from A₀ and S₀ Waves on Submerged Elastic Shells

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Elastic waves propagating on thin shells may be classified similarly to Lamb waves on plates $(A_0, S_0, A_1, S_1, ...)$ and Scholte-Stoneley waves (A) in the external fluid loading. The present study deals with evacuated shells of semi-infinite extent with a uniformly-curved front surface, on which acoustic pulses are incident head-on. The incident signals cause the generation of the shell waves at a critical angle of incidence; these may be observed by their re-radiation into the fluid at the same critical angle. Our demonstration of the re-radiated pulses consists of a numerical evaluation of the incident and re-radiated fields and a visualisation of the corresponding pulses in a tank experiment employing the Schlieren method. While the weaker A wave could not be observed here, we were able to demonstrate by both methods the generation of re-radiated pulses of the A_0 and the S_0 wave, at the same time verifying the value of the critical angle of their generation.

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1. INTRODUCTION

We are studying the generation of elastic waves on waterimmersed, evacuated, semi-infinite thin elastic shells by an acoustic wave which is incident head-on onto the curved front surface of the shell (Fig. 1). Note that the shell wave is created on the curved surface at a critical angle a determined by Cremer's coincidence condition,¹ which is given by $c_p =$ $c_w/\sin a$. Here, c_p is the phase speed of the shell wave and c_w is the sound speed in the ambient water. The critical angle corresponds here to the existence of a coincidence between the shell wave and the trace velocity along the shell of the incident fluid-borne wave. The shell wave can be observed via the head wave,² which it emits back into the water at the same critical angle while propagating along the shell. In the following sections, we study the observation of the head waves both by calculation, and by an experiment that visualises these waves using the Schlieren method. Calculated dispersion curves for the shell wave speeds c_s allow the determination of the critical angle a, which is found to agree with observations.

2. COMPUTATIONAL APPROACH

An existing computer program³ was applied to the scattering of a sound pulse from a shell with the geometry of Fig. 1, with an outer shell radius of r = 14 cm, a thickness of 6.4%, elastic speed of $c_L = 5000$ m/s, and $c_T = 3000$ m/s, and a density of $\rho = 4$ g/cm³ (close to *Al*). An incident sound pulse was considered, as shown in Fig. 2. Computational results were obtained for various times following incidence, a particular result being shown in Fig. 3.

One can see the geometry of the shell towards the right, as well as the various wave fronts. The heavy vertical line is

the front of the incident plane wave, and the heavy semicircular curve pointing towards the left is the pulse specularly reflected from the shell's apex. Inside this specular reflection, one can recognise the symmetric head waves from two shell waves propagating up or down along the curved portion of the shell, which are evidently generated at a critical angle *a* (see Fig. 1). These shell waves can be attributed to the A_0 shell wave.¹⁴ In addition, one observes further to the right the (weaker) head wave from a very fast shell wave that has overtaken the incident wave, and hence can be attributed to the S_0 shell wave.¹ This shell wave can be generated on both the curved and the lateral portion of the scattering object. (The waterborne, very slow *A* or Scholte wave is not visible here.) Below, an experimental demonstration of these head waves is presented.

3. EXPERIMENTAL APPROACH

An experiment was carried out which visualised the head waves optically. This experiment used a water-immersed evacuated glass shell insonified by a pulse emanating from a point source (left-hand side of Fig. 4). This incident pulse has a circular wave front, as does the specularly reflected wave, which is seen as a smaller circle.

However, further to the right one observes the head wave of a shell wave, which can be attributed to the A_0 wave. The faster S_0 head wave, which should be even further to the right, is too weak to be visible here. There should not essentially be any qualitative difference between the head-wave generation in Figs. 3 and 4, as in the latter figure, in which the head wave also propagates along the flat portion of the scattering object.

Since the comparison of the present calculated and experimental results is of a qualitative nature, the shell geometry is not identical in the two cases.