EVALUATION OF ROBUSTNESS OF SOUND FIELD CONTROL WITH A CIRCULAR DOUBLE-LAYER ARRAY OF LOUDSPEAKERS

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The circular double-layer array of loudspeakers has been proven feasible in reproducing an expected sound field inside the array and reducing sound waves propagating outside the array. Usually the performance is evaluated by two criteria concerning the acoustic pressure matching and the acoustic contrast. In this paper, the standard parameter related to the localization cues of human auditory systems, interaural time difference (ITD), is used to evaluate the sound field control result. The robustness of the control methods are fully investigated in the following three conditions, (1) The variation of the head positions, (2) The mismatch between the loudspeakers, and (3) The bias of the loudspeaker positions.

1. Introduction

Sound field reproduction systems attempt to generate a sound field that approximates a desired sound field or try to concentrate sound energy in one or several zones. Among many studies, some have attempted to reproduce desired sound field accurately inside a zone and reduce the sound in other zones.\(^1\)\(^-\)\(^4\) Recently, a method on sound field control has been proposed using a circular double-layer array of loudspeakers whose efficacy have been demonstrated by both simulations and experiments.\(^5\)\(^,\)\(^6\) Although the scattering effect has been examined,\(^7\) the robustness of the system is not yet well addressed.

In this paper, the robustness of the system is investigated under three conditions, (1) the variation of the head positions, (2) the mismatch between the loudspeakers, and (3) the bias of the loudspeaker positions. The efficacy of the reconstructed sound field is quantified by interaural time difference (ITD),\(^8\) and the acoustic focusing ability of the system is quantified by acoustic contrast (AC).\(^9\)

2. Problem formulation

2.1 Statement of the problem

Figure 1 illustrates the circular double-layer array of loudspeakers, the head, the bright zone and the dark zone. The array is composed of \(N\) loudspeakers, located on two circles of radii \(r_L^+\) and \(r_L^-\) regularly. The head and the bright zone \(S_b\) is located inside the loudspeaker array, and the dark zone \(S_d\) is outside the array. The head is a rigid sphere of radius \(r_a\) located at \(r_s = (r_s, \theta_s)\), with "ears" located at diametrically opposed points on the surface of the head. The bright zone is a ring shaped...
region of which the inner radius is \( r_a \) and the outer radius is \( r_b \). The dark zone is a ring shaped region of which the inner radius is \( r_d \) and the width is \( \Delta r_d \), which is expected to be quiet.

Figure 1. The circular double-layer array of loudspeakers, the head, the bright zone and the dark zone.

2.2 Definition of variables

The sound pressure at \( r \) at wavenumber \( k \) generated by the loudspeakers can be expressed as

\[
p(r; k) = \sum_{n=1}^{N} H(r | r_{Ln}) q_n ,
\]

where \( H(r | r_{Ln}) \) is the transfer function between the input signal of the \( n \)th loudspeaker at \( r_{Ln} \) and the sound pressure at \( r \), and \( q_n \) is the complex source strength of the \( n \)th loudspeaker.

The bright zone and the dark zone can be sampled as \( I \) and \( J \) discrete points respectively: \( r_{b1}, r_{b2}, \ldots, r_{bI} \) and \( r_{d1}, r_{d2}, \ldots, r_{dJ} \). Then the sound pressure in the zones can be expressed as

\[
p_b(k) = \begin{bmatrix} p(r_{b1}; k) & \cdots & p(r_{bI}; k) \end{bmatrix}^T, \quad p_d(k) = \begin{bmatrix} p(r_{d1}; k) & \cdots & p(r_{dJ}; k) \end{bmatrix}^T
\]

Similarly, the transfer function and the source strength can be expressed as

\[
H_b = \begin{bmatrix} H(r_{b1} | r_{L1}) & \cdots & H(r_{bI} | r_{LN}) \\ \vdots & \ddots & \vdots \\ H(r_{b1} | r_{L1}) & \cdots & H(r_{bI} | r_{LN}) \end{bmatrix}, \quad H_d = \begin{bmatrix} H(r_{d1} | r_{L1}) & \cdots & H(r_{dI} | r_{LN}) \\ \vdots & \ddots & \vdots \\ H(r_{d1} | r_{L1}) & \cdots & H(r_{dI} | r_{LN}) \end{bmatrix}
\]

\[
q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \end{bmatrix}^T.
\]

Then Eq. (1) can be rewritten in matrix form in different zones as

\[
p_b = H_b q, \quad p_d = H_d q.
\]

2.3 Description of monopoles with consideration of scatterer

The transfer function between the input signal of a monopole at \( r_L = (r_L, \theta_L, \phi_L) \) and the sound pressure at \( r = (r, \theta, \phi) \) with a scatterer can be expressed as

\[
H(r | r_L) = \begin{bmatrix} H(r | r_{L1}) & \cdots & H(r | r_{LN}) \end{bmatrix}
\]
\[
H(\mathbf{r}|\mathbf{r}_L) = \begin{cases} 
4\pi i k \sum_{m=0}^{\infty} \frac{j_m(k r_L)}{h_m^*(k r_L)} h_m^*(k r) \sum_{m=-n}^{n} Y_m^m(\theta_L, \varphi_L)^* Y_m^m(\theta, \varphi), & r_n < r < r_L \smallskip 
4\pi i k \sum_{m=0}^{\infty} \frac{j_m(k r)}{h_m^*(k r)} h_m^*(k r_L) \sum_{m=-n}^{n} Y_m^m(\theta_L, \varphi_L)^* Y_m^m(\theta, \varphi), & r > r_L 
\end{cases}, 
\]  

(6)

where \(j_m(\cdot)\) is the spherical Bessel function, \(j_m^*(\cdot)\) is the first derivative of \(j_m(\cdot)\), \(h_m(\cdot)\) is the spherical Hankel function of the first kind, \(h_m^*(\cdot)\) is the first derivative of \(h_m(\cdot)\), and \(Y_n^m(\cdot)\) is the Laplace spherical harmonics.

### 2.4 Performance indices

The interaural time difference (ITD) are used to check the sound localization ability of the reproduction system, which can be expressed as

\[
\text{ITD} = \Phi / 2\pi f, 
\]

(7)

where \(\Phi\) denotes the phase difference between the ears, \(f\) is the frequency.

The acoustic contrast (AC) proposed by Choi and Kim which denotes the ratio between the average acoustic potential energy density in the bright zone and that in the dark zone can be expressed as

\[
\text{AC} = \frac{J(p_b)}{J(p_d)}. 
\]

(8)

### 2.5 Optimization method

In order to generate accurate sound field in the bright zone and reduce sound energy in the dark zone, the cost function can be expressed as

\[
J(q) = \kappa p_b^H (p_b - p_{b0})^H + (1 - \kappa) \left( p_b - p_{b0} \right)^H \left( p_b - p_{b0} \right), 
\]

(9)

where \(p_{b0}\) is the desired sound field, \(\kappa (0 \leq \kappa < 1)\) is a weighting factor that determines the balance between the potential energy in the dark zone and the reproduction error in the bright zone.

The optimal source strength can be derived by set \(\partial J(q)/\partial q = 0\), which can be written as

\[
q = \left[ \kappa \mathbf{H}_d^H + (1 - \kappa) \mathbf{H}_d^H \mathbf{H}_d \right]^{-1} (1 - \kappa) \mathbf{H}_d^H p_{b0}, 
\]

(10)

### 2.6 Robustness of the reproduction system

#### 2.6.1 The variation of the head positions

For the application of the sound field reproduction system, the listener’s head cannot be assumed to be fixed in the center. If the center of the head moves to \(\mathbf{r}_c\), then \(\mathbf{r}_L\) in Eq. (6) should be replaced by \(\mathbf{r}_c\), and the variation of the head positions can be expressed as

\[
\mathbf{r}_v = (r_v, \delta_v) = \mathbf{r}_c - \mathbf{r}_L. 
\]

(11)

#### 2.6.2 The mismatch between the loudspeakers

When the variation of the loudspeaker characteristics are taken into consideration, the transfer function in Eq. (6) will become

\[
H_{\text{mismatch}}(\mathbf{r}|\mathbf{r}_L) = Z(\mathbf{r}_c) \cdot H(\mathbf{r}|\mathbf{r}_L) = z(\mathbf{r}_c) e^{-j \phi(\mathbf{r}_c)} \cdot H(\mathbf{r}|\mathbf{r}_L), 
\]

(12)
where \( Z(\mathbf{r}_L) \) are the mismatch between the loudspeakers, \( z(\mathbf{r}_L) \) and \( v(\mathbf{r}_L) \) represent the gain and phase mismatch between the loudspeakers respectively.

### 2.6.3 The bias of the loudspeaker positions

Usually the acoustic centers of the loudspeakers are hard to be placed at the ideal positions accurately, then \( \mathbf{r}_L \) in Eq. (6) should be replaced by \( \mathbf{r}_{L,bias} \) which can be written as

\[
\mathbf{r}_{L,bias} = \mathbf{D}(\mathbf{r}_L) \mathbf{r}_L = d(\mathbf{r}_L) e^{-j\phi(\mathbf{r}_L)} \mathbf{r}_L,
\]

where \( D(\mathbf{r}_L) \) are the bias of the loudspeaker positions, \( d(\mathbf{r}_L) \) and \( \mu(\mathbf{r}_L) \) represent the gain and phase bias of the loudspeaker positions respectively.

### 3. Simulations

#### 3.1 Simulation setup

An example is examined in a simple simulation setup. The radius of the double-layer array is \( r_{L-} = 0.9 \) m and \( r_{L+} = 1 \) m, the number of loudspeakers is \( N = 40 \). Considering the size of the listener’s head, the radius of the head is \( r_h = 0.1 \) m and the "ears" is 0.11 m away from the center of the head. The outer radius of the bright zone is \( r_b = 0.2 \) m, the inner radius of the dark zone is \( r_d = 2 \) m, and the width is \( \Delta r_d = 1 \) m. For simplicity, the desired sound field in the bright zone is a plane wave with the amplitude of unity propagating in the negative \( x \) direction.

ITD is a cue for direction perception at low frequency. Here, we just choose 500 Hz to check the ITD performance. The just-noticeable differences are chosen to be 50 \( \mu s \). Hence, the area in which the difference between the ITD of the desired sound field and that of the actual sound field is not larger than the just-noticeable difference is considered to be effective. The frequency range of interest to improve acoustic contrast (AC) is between 100 Hz and 1 kHz.

The robustness of the reproduction system is fully evaluated in the following three conditions. (1) The variation of the head positions, with the range of \( r_v \) and \( \delta_v \) in Eq. (11) are (0,0.1 m) and (0°,180°) respectively. (2) The mismatch between the loudspeakers, with the range of \( z(\mathbf{r}_L) \) and \( v(\mathbf{r}_L) \) in Eq. (12) are (0.6,1.4) and (-5°,5°) respectively. (3) The bias of the loudspeaker positions, with the range of \( d(\mathbf{r}_L) \) and \( \mu(\mathbf{r}_L) \) are Eq. (13) is (0,0.1 m) and (-10°,10°) respectively. The parameters above are generated randomly in the simulation and the results are averaged from 15 trials.

#### 3.2 Results

Figure 2 shows the generated sound field at 200, 500 and 800 Hz. Figure 3 shows the acoustic contrast as a function of frequency. It can be seen that the system can produce the desired sound field in the bright zone accurately and reduce the sound energy in the dark zone at low frequencies efficiently. The acoustic contrast is more than 40 dB below 500 Hz and increases with the increase of the weighting factor \( \kappa \).

![Figure 2](image)

**Figure 2.** Actual sound field (\( \kappa = 0.5 \)) at 200, 500 and 800 Hz from left to right.
Figure 3. Acoustic contrast in the ideal condition.

Figure 4. ITD deviations and effective area with $\kappa = 0.1, 0.5, 0.9$ from left to right.

Figure 5. Average acoustic contrast with the variation of head positions.
The robustness of the reproduction system is summarized as follows.

(1) Influence of variation of head positions

Figure 4 shows the ITD deviations and the effective area in the bright zone and most area in the bright zone is considered to be effective. Figure 5 illustrates the average acoustic contrast with the head positions varying 15 times randomly. Compared to Fig. 3, the acoustic contrast decreases about 3~8 dB, but it is still more than 30 dB below 500 Hz.

(2) Influence of mismatch between loudspeakers

Figure 6 shows the ITD deviations and the effective area in the bright zone with the mismatch between the loudspeakers. As can be seen, most area in the bright zone is invalid with $\kappa = 0.9$, and most area is considered to be effective with $\kappa = 0.1$ and 0.5. Fig. 7 shows the average acoustic contrast with 15 trials. Compared to Fig. 3, the acoustic contrast decreases about 10~25 dB below 500 Hz and is less than 31 dB below 1000 Hz, which means that the mismatch between the loudspeakers causes significant deterioration of the performance.

![Figure 6](image)

**Figure 6.** ITD deviations and effective area with the mismatch between the loudspeakers with $\kappa = 0.1$, 0.5, 0.9 from left to right.

![Figure 7](image)

**Figure 7.** Average acoustic contrast with the mismatch between the loudspeakers.

(3) Influence of bias of loudspeaker positions
Figure 8 illustrates the ITD deviations and the effective area in the bright zone with the bias of the loudspeaker positions. It can be seen that most area is effective for the listener. Figure 9 shows the average acoustic contrast with 15 trials. The acoustic contrast changes a little when compared to Fig. 3. More than 40 dB below 500 Hz is achieved, which means the limited bias of the loudspeaker positions will have little influence on the reproduction performance.

![Figure 8. ITD deviations and effective area with the bias of the loudspeaker positions with $\kappa = 0.1, 0.5, 0.9$ from left to right.](image)

Figure 9. Average acoustic contrast with the bias of the loudspeaker positions.

In order to explain the results of the robustness of the reproduction system, Frobenius norm of the difference between the ideal transfer function matrix and the actual transfer function matrix are calculated, which can be expressed as

$$G = \| H_{\text{actual}} - H_{\text{ideal}} \|_F \quad (14)$$

The average $G$ caused by the variation of the head positions, the mismatch between the loudspeakers and the bias of the loudspeaker positions are 117.7, 1157.1 and 455.8 respectively. This clearly explains that the mismatch between the loudspeakers will deteriorate the performance seriously. On the contrary, the reproduction system is robust to the variation of the head positions or the bias of the loudspeaker positions.
4. Conclusion

This paper has evaluated the robustness of the sound field control with a circular double-layer array of loudspeakers. The reproduction system is robust to the variation of the head positions and the bias of the loudspeaker positions. However, the mismatch between the loudspeakers will deteriorate the performance of the reproduction system seriously.

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