AN ACTIVE DISTURBANCE REJECTION CONTROL APPROACH IN A VIBRATING MECHANICAL SYSTEM

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This paper describes the application of an active disturbance rejection control approach in a linear mass-spring-damper vibrating mechanical system. Robustness is considered against parametric uncertainties, exogenous forces and input unmodeled dynamics. Internal and external uncertainties are lumped in a disturbance input which is estimated on-line and directly rejected by the controller. The presented control approach can be extended for differentially flat, linear or nonlinear, multiple degrees-of-freedom vibrating mechanical systems. Some simulations results show the efficient performance of the proposed control scheme and the effectiveness of the disturbance estimation.

1. Introduction

Unknown exogenous forces, parametric uncertainty and unmodeled dynamics are some challenging problems in active vibration control design. Active disturbance rejection control methodology offers an alternative choice to face these practical issues (see [1-3] and references therein). This paper describes the application of an active disturbance rejection control approach in a perturbed linear mass-spring-damper vibrating mechanical system of two degrees of freedom. The structural property of differential flatness exhibited by the mechanical system is used in the synthesis of the proposed feedforward and feedback tracking control scheme. Robustness is also considered against parametric uncertainties, exogenous forces and unmodeled dynamics. Internal and external uncertainties are lumped in a disturbance input which is estimated on-line by an extended state observer, and directly rejected by the controller.

The design methodology of so-called GPI observers is applied for the synthesis of the presented high-gain estimation scheme for disturbances and time derivatives of the output variable [4]. In our observer-based control design approach, the unknown bounded disturbance is locally reconstructed by using a Taylor polynomial family of forth order. The performance of the control scheme is verified for a two degrees-of-freedom underactuated linear vibrating mechanical system perturbed by a secondary uncertain vibration system. Some simulation results are included to show the effectiveness of the proposed active disturbance rejection control scheme and the acceptable estimation of the disturbance input and time derivatives up to third order of the output position signal. The presented control approach can be extended for differentially flat, linear or nonlinear, multiple-degrees-of-freedom vibrating mechanical systems.
2. Mass-spring-damper mechanical system

Consider the under-actuated linear mass-spring-damper mechanical system shown in Fig. 1. Suppose that dynamics of the primary system, constituted of the first two masses, sprigs and dampers, is only known. Moreover, assume that an uncertain secondary mass-spring-damper system is coupled to the second mass. Therefore, the primary mechanical system is perturbed by the unknown state dependent spring force \( f = k_3(x_3 - x_2) \).

The mathematical model of the mechanical system is described by

\[
\begin{align*}
    m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 &= k_2(x_2 - x_1) + u \\
    m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2(x_2 - x_1) &= k_3(x_3 - x_2) \\
    \vdots \\
    m_{n-1} \ddot{x}_{n-1} + c_{n-1} \dot{x}_{n-1} + k_{n-1}(x_{n-1} - x_{n-2}) &= k_n(x_n - x_{n-1}) \\
    m_n \ddot{x}_n + c_n \dot{x}_n + k_n(x_n - x_{n-1}) &= 0 \\
\end{align*}
\]

where \( x_i, i = 1, 2, \ldots, n \), are mass positions, \( u \) represents the active force control input, \( y = x_2 \) is the position output variable to be controlled, and \( m_i, k_i \) and \( c_i \) are the mass, stiffness and damping parameters.

![mass-spring-damper mechanical system](image)

Defining the state variables as \( z_1 = x_1, z_2 = \dot{x}_1, z_3 = x_2 \) and \( z_4 = \dot{x}_2 \), one obtains the state space description for the primary system

\[
\begin{align*}
    \dot{z}_1 &= z_2 \\
    \dot{z}_2 &= -k_1 k_2/m_1 z_1 - c_1 m_1^{-1} z_2 + k_2/m_1 z_3 + 1/m_1 u \\
    \dot{z}_3 &= z_4 \\
    \dot{z}_4 &= k_1/m_2 z_1 - k_2/m_2 z_3 - c_2/m_2 z_4 + 1/m_2 f \\
    y &= z_3 \\
\end{align*}
\]

Model (2) exhibits the structural property of differential flatness. State and control variables can be then expressed in terms of the flat output \( y \) and a finite number of its time derivatives [5]. From \( y \) and its time derivatives up to fourth order, unperturbed differential parameterization results in

\[
\begin{align*}
    z_1 &= y + \frac{c_2}{k_2} \dot{y} + \frac{m_2}{k_2} \ddot{y} \\
    z_2 &= \dot{y} + \frac{c_2}{k_2} \ddot{y} + \frac{m_2}{k_2} y^{(3)} \\
    z_3 &= y \\
    z_4 &= \dot{y} \\
    u &= \frac{1}{b} \left( y^{(4)} + a_3 y^{(3)} + a_2 \ddot{y} + a_1 \dot{y} + a_0 y \right) \\
\end{align*}
\]
Thence, dynamics of the flat output $y$ can be described by the input-output model

$$y^{(4)} = bu - a_0 y - a_1 y' - a_2 y'' - a_3 y^{(3)} + \phi$$

with

$$
\begin{align*}
a_3 &= \frac{c_1}{m_1} + \frac{c_2}{m_2} \\
a_2 &= \frac{k_1}{m_1} + \frac{k_2}{m_2} + \frac{k_3}{m_1 m_2} + \frac{c_1 c_2}{m_1 m_2} \\
a_1 &= \frac{c_2 k_1}{m_1 m_2} + \frac{c_2 k_2}{m_1 m_2} \\
a_0 &= \frac{k_3}{m_1 m_2} \\
b &= \frac{k_1}{m_1 m_2}
\end{align*}
$$

In the transformed differentially flat mechanical system dynamics, the term $\phi$ has been included to consider the influence of the disturbance force $f$ as well as small parametric uncertainties. Hence, one can get from (4) the differential flatness controller for tracking tasks of some reference position trajectory $y^*(t)$

$$u = \frac{1}{b} \left[ y^{(4)*} - \alpha_3 (y^{(3)} - y^{(3)*}) - \alpha_2 (y' - y'^*) - \alpha_1 (y - y^*) - \alpha_0 (y - y^*) - \xi \right]$$

with

$$\xi = - a_0 y - a_1 y' - a_2 y'' - a_3 y^{(3)} + \phi$$

Closed loop tracking error dynamics, $e = y - y^*$, is then given by

$$e^{(4)} + \alpha_3 e^{(3)} + \alpha_2 \dot{e} + \alpha_1 e + \alpha_0 e = 0$$

Thus, controller design parameters $\alpha_j$, $j = 0, \ldots, 3$, should be selected so that the characteristic polynomial associated with (7) is Hurwitz. In this way, asymptotic convergence of the error $e$ to zero is guaranteed. Hence, reference trajectory tracking is verified, $y \to y^*$.

Nevertheless, controller (5) demands measurements of position, velocity, acceleration, jerk and disturbances. In addition, positive parameter constants $a_j$ and the control input gain $b$ must be accurately known. Clearly, these restrictions can limit the applicability of the controller. Therefore, in the next section, an active disturbance rejection (ADR) control approach is proposed to deal with those practical difficulties.

### 3. An active disturbance rejection control scheme

Consider that the position output variable $y$ is only available for the ADR control synthesis. Moreover, the control input gain of the system, $b$, is solely known. In the ADR control approach, $\xi$ is considered as an unknown bounded disturbance input, which is properly estimated and rejected on-line. Notice that parametric uncertainty and exogenous disturbances can be also included in $\xi$.

For synthesis of the disturbance estimation scheme, disturbance input $\xi$ is locally approximated by a family of Taylor polynomials of fourth degree around a given time instant $t_0$ as (see [6] and references therein)

$$\xi(t) \approx \sum_{i=0}^{4} \lambda_i (t - t_0)^i$$
where coefficients $\lambda_i$ are unknown.

Hence, we obtain the extended state model

$$\begin{align*}
\dot{y}_j &= y_{j+1} & j &= 1, \ldots, 3 \\
\dot{y}_4 &= bu + \xi_1 \\
\dot{\xi}_j &= \xi_{k+1} & k &= 1, \ldots, 4 \\
\dot{\xi}_5 &= \delta
\end{align*}$$

(9)

where $y_1 = y$, $y_2 = \dot{y}$, $y_3 = \ddot{y}$, $y_4 = y^{(3)}$, $\xi_1 = \xi$, $\xi_2 = \dot{\xi}$, ..., $\xi_5 = \xi^{(4)}$, and $\delta$ represents an unknown bounded perturbation including the influence of small residual terms of the truncated Taylor polynomial expansion (8).

Then, from (9) we propose the following high-gain state observer for asymptotic estimation of $\xi$ and time derivatives up to third order for $y$:

$$\begin{align*}
\hat{\dot{y}}_j &= \hat{y}_{j+1} + \beta_0(y_1 - \hat{y}_1) & j &= 1, \ldots, 3 \\
\hat{\dot{y}}_4 &= bu + \hat{\xi}_1 + \beta_1(y_1 - \hat{y}_1) \\
\hat{\dot{\xi}}_k &= \hat{\xi}_{k+1} + \beta_2(y_1 - \hat{y}_1) & k &= 1, \ldots, 4 \\
\hat{\dot{\xi}}_5 &= \beta_0(y_1 - \hat{y}_1)
\end{align*}$$

(10)

Estimation error is then governed by

$$\begin{align*}
\dot{e}_j &= -\beta_0 e_1 + e_{j+1} & j &= 1, \ldots, 3 \\
\dot{e}_4 &= -\beta_1 e_1 + e_{\xi_1} \\
\dot{e}_{\xi_k} &= -\beta_2 e_1 + e_{\xi_{k+1}} & k &= 1, \ldots, 4 \\
\dot{e}_{\xi_5} &= -\beta_0 e_1 + \delta
\end{align*}$$

(11)

where $e_j = y_j - \hat{y}_i$, $j = 1, \ldots, 4$, and $e_{\xi_k} = \xi_k - \hat{\xi}_k$, $k = 1, \ldots, 5$.

The characteristic polynomial of the observation error dynamics is then given by

$$P_O(s) = s^9 + \beta_8 s^8 + \cdots + \beta_1 s + \beta_0$$

(12)

Thus, the design parameters, $\beta_j$, $j = 0, 1, \ldots, 8$, should selected so that the characteristic polynomial (12) be a Hurwitz polynomial. Moreover, observer’s natural frequencies should be sufficiently high in order to uncouple the control dynamics from observer. Therefore, the observation error is bounded for any $\delta$ bounded.

Hence, the estimated signals obtained from extended state observer (10) can be used in the implementation of the differential flatness controller (5), resulting in the ADR control scheme

$$u = \frac{1}{b} \left[ y^{(4)*} - \alpha_3(\bar{y}_4 - y^{(3)*}) - \alpha_2(\bar{y}_3 - \bar{y}^*) - \alpha_1(\bar{y}_2 - \bar{y}^*) - \alpha_0(y - y^*) - \dot{\xi} \right]$$

(13)

4. Simulation results

The performance of the extended state observer-based control scheme (13) was verified for a mechanical system with three degrees of freedom characterized by the set of parameters given in Table I.

The design parameters of the controller were selected to have the following characteristic polynomial for the closed loop tracking dynamics:

$$P_c(s) = (s^2 + 2\zeta_n\omega_n s + \omega_n^2)^2$$

(14)
Table 1: Parameters of the 3 DOF mass-spring-damper system.

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<td>$m_1$</td>
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<td>$c_1$</td>
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<td>$m_2$</td>
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<td>$k_3$</td>
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with natural frequency $\omega_{nc} = 10$ rad/s and damping ratio $\zeta_c = 0.7071$.

The characteristic polynomial for the estimation error dynamics was set to be of the form

$$P_o(s) = (s + p_o)(s^2 + 2\zeta_0\omega_{no}s + \omega_{no}^2)^4$$

with $p_o = 200$ rad/s, $\omega_{no} = 200$ rad/s and $\zeta_0 = 0.7071$.

The motion profile $y^*$ planned for the mechanical system was conveniently described by a Bézier interpolation polynomial [7]. The control objective was specified to smoothly transfer the output variable $y$ from the initial rest position $y_i = 0$ to the final equilibrium position $y_f = 0.01$ in 5 s.

In this case study, the mechanical system was perturbed by un-modeled dynamics associated to the unknown secondary mechanical system $(m_3, c_3, k_3)$. The satisfactory tracking of the reference position trajectory can be verified in Fig. 2.

Fig. 3 show the applied control force $u$ using estimated signals provided by the high-gain state observer (10). The acceptable estimation of the disturbance input and time derivatives up to third order of the output variable $y$ are depicted in Figs. 4 and 5. Therefore, the presented ADR control approach represents an alternative choice for the controller synthesis for under-actuated perturbed linear mass-spring-damper mechanical systems of two degrees of freedom employing only measurements of the position output variable.

5. Conclusions

In this paper, an ADR control scheme has been proposed for robust reference position trajectory tracking tasks for a topology of under-actuated linear mass-spring-damper mechanical systems of two degrees of freedom subjected to disturbances due to un-modeled dynamics, parametric uncertainties and, possibly, exogenous forces. State dependent disturbances were induced by couplings of the system with another unknown mass-spring-damper system. Internal and external uncertainties were lumped in a disturbance input which was estimated on-line and directly rejected by the controller. The disturbance input was locally approximated by a family of Taylor polynomials of forth degree. Thus, an extended state mathematical model was employed for the synthesis of a high-gain state observer for asymptotic estimation of the disturbance input and some time derivatives of the output position variable required for control implementation. Effectiveness of the ADR control scheme was verified for a mechanical system with three degrees of freedom. The acceptable estimation of the unknown signals was shown. Therefore, one can conclude that the ADR control approach represents a good choice for the controller synthesis for under-actuated perturbed linear mass-spring-damper systems of two degrees of freedom.

REFERENCES


Figure 2: Closed loop position responses of the perturbed mechanical system.
Figure 3: Control force using estimation of disturbance and derivatives of the output signal.

Figure 4: Estimation of disturbance signal using fourth order Taylor polynomials.
Figure 5: Estimation of time derivatives up to third order of the output variable.